AN ANALYSIS OF AGGREGATED EFFECTIVENESS FOR INDIRECT ARTILLERY FIRE ON FIXED TARGETS

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AN ANALYSIS OF AGGREGATED EFFECTIVENESS

FOR INDIRECT ARTILLERY FIRE ON FIXED TARGETS

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SUMMARY

This research addresses the problem of aggregated effectiveness of a field artillery battery using indirect fire on fixed targets. The analysis was made using the four weapons configurations of 2, 3, 4 and 6 guns per volley as treatment levels. The research was limited in scope to the 155mm Self-Propelled Howitzer, M109A1, engaging circular type targets with unadjusted and adjusted fire.

A Monte Carlo simulation model developed by the Braddock, Dunn and McDonald Services Company was utilized for computational purposes to generate output data. The measures of effectiveness were rounds on target, the average miss distance of a volley and the average radial error of rounds in a volley. The analysis of variance (ANOVA) and Newman-Keuls range tests were made on the measures to determine statistical levels of significance.

The results indicated that the four-gun unit operating independently was the most effective system. The model proved useful in representing artillery systems, in analyzing the alternative adjustment technique under the limited rounds constraint and for updating the JMEM tables for artillery effects.

CHAPTER I

INTRODUCTION

Description of the Problem

Since World War II the effectiveness of field artillery fire has been based on the doctrine of massing of fires from many guns and with few limitations on numbers of rounds employed. The total effectiveness was perceived as monotonically increasing as the number of pieces increased and was based on the precision of each weapon utilizing a common acquisition and control system. Recently technological advances in sensory, data processing and warhead devices now offer the potential for more accurate and timely means of delivery and more efficient allocation of rounds from smaller numbers of weapons to accomplish the same level of effectiveness.

The current system in the field artillery has several weaknesses which reduce the effectiveness which can be achieved. First, only one location of the battery is established in the fire direction procedures and this is used to determine the range to the target for all six weapons in the battery. Second, usually one aimpoint is used for the weapons to fire upon and the dispersion pattern of the weapons on the ground determined where the rounds would impact. Third, there are no realistic guidelines to determine how many rounds need to be fired to achieve the desired destruction. Tabulated effects manuals such as the Joint Munitions Effects Manual (JMEM) indicate a large number of battery volleys as being required to achieve an acceptable level of destruction. These established requirements would exhaust a unit's ammunition supply in a very short period of time on only a few targets. Fourth, the traditional approach of using two weapons in adjustment of fire missions may not be as effective as other configurations which have never been considered.

The current fire direction system has the Field Artillery Digital Analog Computer (FADAC) which processes the input information of the weapons and the targets to determine firing data to achieve rounds on target. The system is slow and is becoming outdated in its operational capabilities. A single weapon is located with any type of accuracy in the system while the other weapons are approximately located in position. A new system was needed to increase mission effectiveness in the target area, increase battlefield survivability through modern techniques and to provide for independent or autonomous battery operations. New technology has introduced a Battery Computer System (BCS) which provides these additional capabilities and the flexibility into the fire support role of the field artillery. The new system will locate each weapon in the battery by exact coordinates and will compute separate data for each weapon and use multiple aimpoints to achieve better coverage of the target. Several alternative concepts exist for employment of the BCS which allow more flexibility and give greater effective firepower for the supported unit.

The Chief of Staff of the U. S. Army has been briefed recently concerning a proposed restructure of the direct support artillery units which involves an eight-gun battery instead of the current six-gun configuration. The eight-gun battery will be capable of operation under three concepts which include: 1) an eight-gun unit with one BCS; 2) two four-gun units with one BCS; and 3) two four-gun units with two BCS control systems. The latter system will enable the two separate units to have autonomous operation [11].

There are several justifications for this increase in the number of weapons per battery. The primary purpose is to increase the firepower of the unit and to allow more weapons to be available when maintenance prohibits the full authorized allocation of weapons to be operational. The concept of splitting a battery into smaller units is very critical in enhancing the survivability of the artillery weapons. If the smaller units can provide the same or better fire support than the current six or proposed eight gun systems, it would be more feasible and appropriate to use the smaller units as the revised doctrine.

The most appropriate question which arises when viewing the capabilities of the new system is how many weapons should be used in these units to engage a target of opportunity. A related question is once it is determined how many rounds are needed to neutralize the target, how are these rounds delivered by different configurations of weapons.

The current doctrine specifies that one weapon is used in the adjustment for a precision registration mission and two guns in the center of the battery are used for the adjusted fire mission. There have been numerous studies done to investigate the effectiveness of these concepts but very little has been done to consider the effect of various combinations of weapons in a battery firing subsequent volleys on a target. The evaluation of three, four or six guns per volley firing may prove significant in achieving better and quicker results on target destruction. This paper is directed at the problem of aggregated effectiveness of two through six guns to determine if there is a real difference in using multiple weapon configurations.

The importance of investigating the effects of multiple configurations is to enable the field artillery to consider the alternative operational systems which may be employed in the operation of the BCS and to adopt the system for field use. Most of the analytical studies and computer simulations involve the battalion or group of battalions in the massed fire mode as evaluated by The RAND Corporation [44] and Vector Research Incorporated [54]. Relatively little attention has been given to the problem of gun allocations within the battery.

Objective of the Research

The objective of this research is to investigate the aggregated level of performance of a United States Army Field Artillery Battery with the capabilities of a Battery Computer System and to provide a basis of selection of the number of weapons which should be used to effectively engage specific types of fixed targets. The investigation includes the analysis of the impact of increasing numbers of pieces to determine the total unit effect and the demonstration of the usefulness of an improved model which provides multi-piece fire for the field artillery.

Scope for the Research

The analysis of the system will include the inherent characteristics of the 155mm Self-Propelled Howitzer, M109A1, in a Field Artillery Battery. The errors associated with the range, deflection and circular probable distances and the specific target description will be used in determining the measures of effectiveness (MOE). The measures to be considered include area coverage, rounds on target, mean point of impact (MPI) of a group of rounds and mean radial error (MRE). The data to be analyzed was generated by an experimental design utilizing the simulation model to represent the weapons system.

In this research the configurations of weapons are regarded as the different treatment levels and their related responses are tabulated and displayed by indicating the effectiveness versus the number of pieces for a given operational doctrine. The analysis describes the most efficient number of pieces necessary to engage specific targets to render them ineffective.

CHAPTER II

CURRENT ARTILLERY DOCTRINE

The Field Artillery System

The mission of the field artillery is to provide continuous and timely fire support to the force commander by destroying or neutralizing in priority, those targets that jeopardize the accomplishment of his mission. This support is achieved by the field artillery organization which consists of all of those elements that are necessary to obtain the desired rounds on the target. These elements include: the weapons, target acquisition, survey, ballistic meteorology, communication mobility both through the air and on the surface, logistics, fire control and coordination, automatic data processing, ammunition, organization and employment [17].

Organization

The battery is organized to operate independently in a sector of the battlefield supporting a maneuver force. The normal battery has six major weapon systems or howitzers capable of indirect and direct fire. The system being analyzed in this paper is the 155mm, Self-Propelled Howitzer, M109A1, a medium range weapon with a maximum range of 18,200 meters.

The battery has three forward observer (FO) teams assigned which are located with the maneuver force for target acquisition and engagement. When the FO is established in a position, he reports his location and status and prepares for his mission. He locates prospective targets based on the terrain, the objectives of the maneuver force and the most likely areas where artillery fire will be needed.

The Fire Direction Center (FDC) acts as the nucleus of the battery and coordinates all of the operations of the elements. The FDC processes all the necessary information to become operational and must maintain communications with the key elements to accomplish the mission.

Fire Mission Procedure

There are two types of fire missions to be considered in the artillery effects analysis, unadjusted fire and adjusted fire. Recently unadjusted or unobserved fire has increased in military importance since offensive actions may include preparations or prearranged fires in conjunction with an attack and may not allow the observer the opportunity of knowing where the rounds impacted. When observed and adjusted fire is employed, there is immediate feedback as to the effectiveness of the fire on the target. Adjustments may be made to allow subsequent rounds to be placed closer to the target and assure better destruction. Unadjusted fire is defined as sequential fire on the same aimpoint. Adjusted fire is defined as fire that is corrected on a sequence of aiming points for a moving target or fire that compensates for biases in locating a stationary target.

The basic procedure for completing a fire mission has been established through doctrine and training. The forward observer initiates the fire mission by acquiring a target either through direct observation or from the supported maneuver unit. He processes the necessary information to

.e target for the artillery battery by several methods, such as ɔordinates, polar plot from his location or a shift from a previously ,vn location. He transmits his message for fire to the fire direction ,enter which processes the information by means of a digital analog computer. The information is translated and processed into firing data for the weapons to orient in the proper direction and elevation. When the rounds are fired for an adjust fire mission, normally the center two weapons fire to allow the observer to make an assessment of how close the rounds are to his estimated target location. If necessary, he makes corrections and transmits them to the battery for subsequent adjustments. Once the observer determines the rounds have sufficiently bracketed the target and he is within 50 meters of the desired adjusting point, he requests fire for effect which normally will involve all weapons in the battery firing on the target.

When initial fire for effect rounds are desired, the observer requests the required rounds to be fired immediately with all available weapons. This procedure is often desirable since surprise is achieved over the enemy by an unexpected volume of fire in a short period of time and the enemy has very little reaction time as in the adjust fire mission. There must exist some basic conditions to ensure an initial fire for effect mission is successful. The target location must be accurate, the observer's location must be known for the polar plot method, and the weapons should have fired a previous registration on a known point to have current meteorological and velocity error corrections and weapons position corrections in the system. When these conditions are satisfied the observer has a relatively high assurance that the fire mission will be successful and the rounds will impact on the ground where desired within normal dispersion [16].

The determination of how many rounds to fire has been based on the situation and on the fire direction officer who specifies the number of rounds and volleys to be fired. The forward observer has the responsibility to terminate the fire mission when he determines that the mission was accomplished in a satisfactory manner. If he observes deviations in the fire for effect rounds and does not assess the target as being neutralized, he may request additional rounds as necessary.

The precision registration mission mentioned earlier is a very necessary requirement for the field artillery. This registration is traditionally fired as the initial mission in the new location and is used to accurately determine weather and weapons corrections to be used in subsequent firing from that location. One base piece is used which is representative of the mean velocity of the battery and usually is the best weapon in the unit. A typical registration involves approximately 16 to 24 rounds each fired singly and adjusted around the target to achieve a precise measure of error and a preponderance of effects to yield range and deflection corrections.

Due to the large amount of time required for such a mission and the number of rounds expended, the Field Artillery School at Fort Sill, Oklahoma conducted a study of two guns firing a precision registration and compared the results to the one gun approach [10]. Several conclusions resulted from this analysis as follows:

(1) Fewer volleys were fired with the two gun approach;

(2) One gun required less mission time than did the two guns,

mainly due to having to prepare two weapons each time;

(3) One gun adjustment expended slightly more than half the rounds used in two gun adjustment;

(4) Multiple bursts are more favorable to accurate spottings of rounds, especially for height of burst with fuze time;

(5) Two guns would be better for adjustment of fuze time but the center weapon of the battery is better for area fire adjustment of fuze quick.

It should be noted that the testing was not done under adverse conditions (i.e., poor visibility, terrain and observer location) and professional experience indicated that multiple bursts facilitated more accurate spotting and less judgment by the observer. No ammunition constraint was placed on this analysis as to the number of rounds fired. Based on the study the only recommended change was that two guns be used in fuze time adjustment and in conditions of poor visibility.

The Acquisition Techniques

The FO has several sources of information about enemy targets but his primary means is direct observation. Critical factors which influence how well he is able to detect, locate and engage targets are his observation post, the terrain, the weather and how well his equipment functions. The two basic devices used to acquire the target are binoculars and the laser range finder. Both devices have inherent accuracies associated with their use and these characteristic standard deviations have been used to describe the target location error in this paper.

Distribution of Fire

There are three basic types of delivery techniques for rounds being fired on a target. These are a parallel sheaf, a converging sheaf and an open sheaf. Normally a parallel sheaf is used in which all weapons fire the same data and the trajectories of the rounds will be parallel causing the impact points to correspond to the width and depth dimensions of the battery formation as shown in Figure 1. A converging sheaf is employed for a point or small target to concentrate the number of rounds in a small location and the trajectories converge to one point as in Figure 2. An open sheaf describes a diverging pattern of rounds where a wider spread of rounds is used to cover a much larger area as in Figure 3.

The actual number of rounds fired on specific targets is constrained by factors such as available ammunition, priority and schedule of fires, the time available to engage a target and how effective the rounds were on the target. If it can be determined how many rounds to fire and in what configuration should these rounds be delivered, the result would be more efficient fire support, conservation of ammunition and more effective fire planning.

Errors Analysis

There are numerous possibilities for errors to occur in the processing cycle for a fire support mission. It is important to be aware of these systems probable errors, firing table probable error and location probable error to better understand the complexity of the system being analyzed. Definitions of accuracy and precision are in Appendix A.

Initially the forward observer is the source of most of the error. He must accurately determine his location in order to report where he is. He must estimate the target location on the ground by the range and

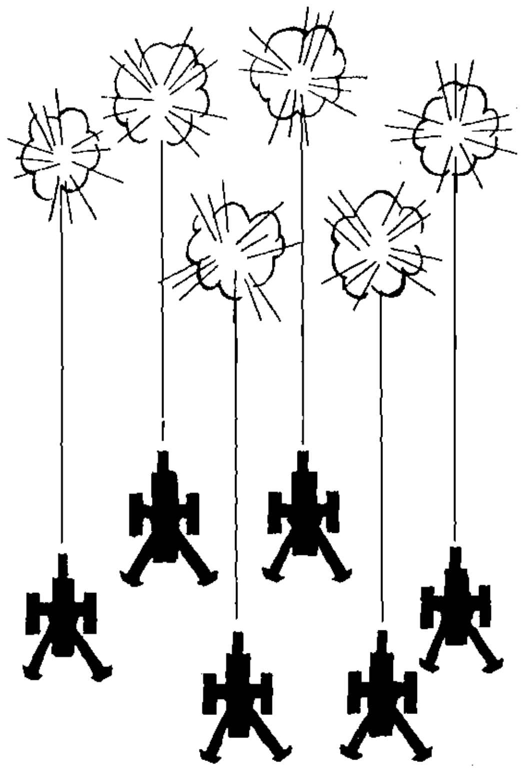


Figure 1. Parallel Sheaf.

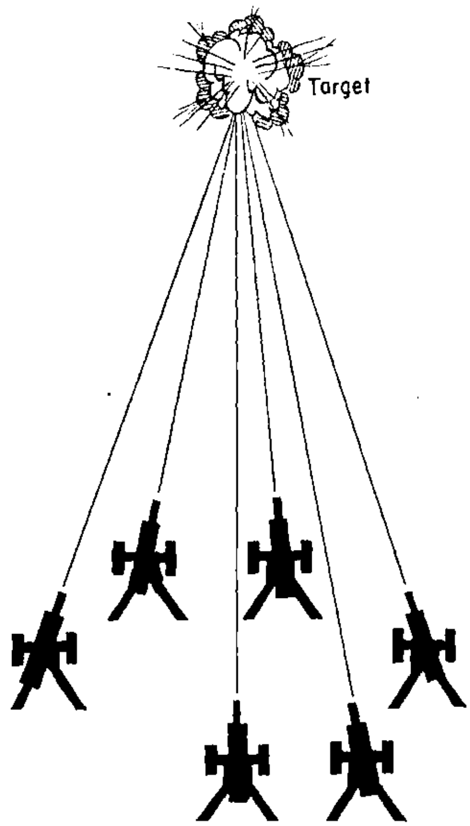


Figure 2. Converging Sheaf.

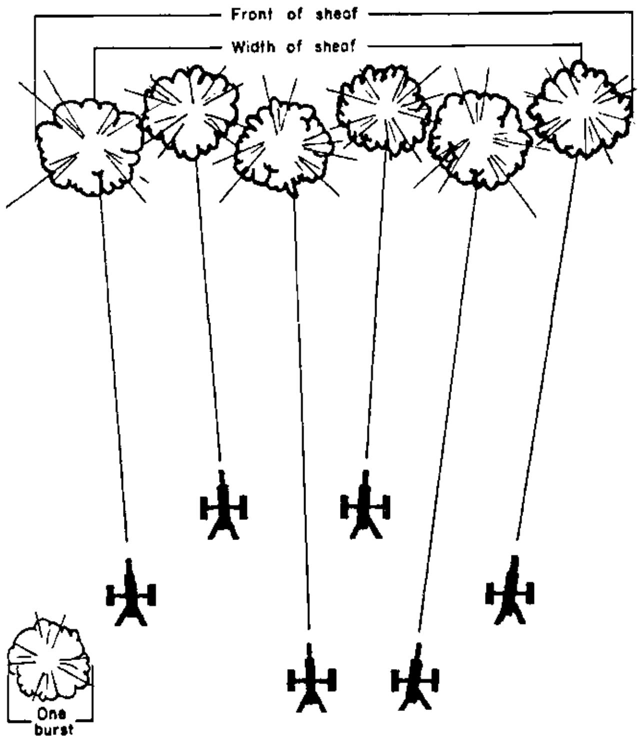


Figure 3. Open Sheaf.

azimuth, by grid coordinates from a map, or by a shift from a known point. His calculations may be in error in processing what he sees. His transmission of the fire mission may be incorrect due to transposition of the numbers or improper identification on the map. Once the fire direction center receives the mission, they must translate the request into firing data for the weapons. The Field Artillery Digital Analog Computer (FADAC) takes into account the weather, propellant temperature, projectile weight, muzzle velocity of the weapons, drift corrections for the flights and many other known effects. When there is a change in these conditions, the expected performance of the projectile may change greatly.

When the weapon is fired, there are a series of factors which could lead to variations on the piece itself. The stabilized position of the weapon could be disrupted. The wear of components or slack involved in mechanisms controlling the platform contribute to error. Other critical areas include tube wear, position of the round in the tube when prepared for firing and the amount of play in the fire control devices which set the data. All of these factors contribute to the system delivering erratic rounds.

Once the mission has made the transition through these stages, and the projectile approaches a point of impact, the actual location of the burst is subject to the general laws of probability. Since the majority of projectiles can be expected to impact within eight probable errors in both range and deflection, the ability to predict exactly where the rounds will land is very limited.

One can be assured that the total pattern of a large number of bursts is elliptical in shape and a rectangle normally is drawn to include the complete distribution of rounds. This precision error is described by probable error in range and in deflection.

Another method of describing how errors are considered in analysis of artillery systems is to extract the definition from the Field Artillery Cannon Gunnery Field Manual 6-40 [16].

If a number of rounds of the same caliber and the same lot are fired from the same weapon, the rounds will not fall at a single point but will be scattered in a pattern of bursts. The natural phenomena of chance is called precision. The array of the bursts on the ground is the dispersion pattern. The points of impact of the projectiles will be scattered both laterally (deflection) and in depth (range). Dispersion is the result of minor variations of many elements from round to round and must not be confused with variation in point of impact caused by mistakes or constant errors. Mistakes can be eliminated and constant errors compensated for by adjustments. These inherent errors are caused in part by:

(1) Condition in the bore - propellant charge, weight, moisture, temperature, powder grains, variation in ramming, bore temperature from round to round.

(2) Condition in the carriage - affected by play looseness in mechanisms, by physical limits on precision.

(3) Condition during flight - weather, drift of projectile.

Mean Point of Impact

When rounds from single volleys impact on the ground, the measure of how close they are to the target is called the mean point of impact (MPI). This mean point is the intersection of two lines, one perpendicular to the line of fire when one-half the rounds fall on each side and one parallel to the line of fire when rounds are divided into two equal amounts. The MPI is usually different with each set of rounds fired. The measure however is quite useful in analyzing closeness to the target.

Effects Analysis

The Field Artillery analysts use many techniques to obtain the approximation of artillery damage to targets both by nuclear and non-nuclear weapons. Analytical methods were used for a long time before the age of computer technology. One typical study which was representative of the analytic method is considered as important to the study of artillery effects as any other and is described below.

The procedure for calculating the number of volleys required to produce a desired effect was developed as a result of a study of effects analysis methods used by the U. S. Army Artillery and Missile School Office of Combat Development and Doctrine in 1962 [51]. This procedure was based on statistical analysis of artillery effects.

The procedure was felt to be quite valid and reliable even though it was based on very limited data. Systems probable errors and target location errors are the main points of concern in the procedure. The expected effects from predicted fire can never by calculated until these two areas are thoroughly studied.

There are several definitions which should be included in this discussion to establish a solid base. The term "effects", or damage level, or neutralization, is an expression of the probability that a man or unit of materiel located at any point in a target area will be rendered ineffective as the result of a volley of artillery aimed at the center of the target area. Thirty percent effects or neutrality refers to the 30% probability that any one man becomes a casualty, or 30% effects may indicate that, on the average, 30% of the personnel in the target area will become casualties.

In order to develop a mathematical analog to the effects of artillery firing, the basic assumptions were made of personnel or target units uniformly distributed in the target area and that the artillery rounds will be normally distributed around the impact point. The study of effects was based on an equation derived from a statistical analysis using these assumptions and was represented as follows:

$$f_n = 1 - \exp(-N_n \, \overline{P} \, \phi \, A_L/A_T) \tag{1.1}$$

where

f_n = the fraction of personnel in the target area when the nth volley hits who will become casualties.

$\overline{P}$ = the probability that rounds fired will land in the target area.

N_n = the number of rounds fired in the nth volley.

A_L = the maximum lethal area of a single round which can be achieved by the caliber weapon used.

A_T = the area of the target.

ϕ = the fraction of the lethal area which can be expected to have effect in the volley due to degree of protection.

The negative expression shown as the exponent of the natural logarithm is the ratio of the total lethal area achieved by a given volley to the total target area. This is basically the neutralization which can be expected during a volley. If a lethal area achieved is one-fourth the size of the target, 25% neutralization should result. However, to compensate for overlap, since it is hard to kill a target portion which has already been killed, the exponential form is introduced. N_nA_L is the maximum lethal area which can be expected for the n^{th} volley and this is reduced by $\overline{P}$ which takes into account the possibility of rounds landing outside the target area. The factor ϕ accounts for the degree of protection of the target and also reduced the lethality.

The probability that rounds fired land in the target area, $\overline{P}$, is dependent on three errors. These are a systems probable error, a firing table probable error, and a location probable error.

Systems errors are those which are induced by methods of calculating, transmitting, and applying the firing data to the weapon. With the gun direction computer and the accurate laying device, the aiming circle, the systems errors become negligible. In this approach the systems error accordingly was assumed to be zero.

The firing table probable error accounts for range and deflection dispersion due to inherent faults in the weapon and the rounds. This might include tube wear, propellant temperature, weight, etc. If a constant angle of fall of the projectile is assumed, it is found that the firing table errors in range and deflection are nearly constant [51].

In using this damage function to generate a table of casualty values for the 155mm howitzer, it was necessary to use values which were approximated to preclude using classified information relating to lethal areas. Since the bursting radius of fragments from the 155mm projectile is 25 meters, this radius was used to approximate the lethal area of a round.

The probability of rounds landing on target for the 155mm is 0.91 in this analysis.

Fractional casualties expected values were generated from the formulation above for several target radii at increasing numbers of weapons firing and are shown in Table 1. The interpretation is that to achieve a desired level of casualties on a certain target one must have n rounds impact on the target area. For example, for a target of 50 meter radius, two rounds achieve a 0.366 fraction of casualties or covers 36.6% of the target area. It should be noted that this is only an approximation based on the input parameters.

Adjustment Procedures

The FO causes the mean point of impact to be placed on, or sufficiently close to, the target by making appropriate corrections during the adjustment. From his spottings, the observer determines deviation and range corrections in meters and transmits these corrections in sequence to the FDC to bring the bursts to the desired point. The three basic bias correction schemes are: 1) establishing a bracket of the adjusting point by overcorrecting the rounds; 2) a halving technique where one-half the correction is used to creep onto the target; and 3) a total bias technique where the rounds are corrected to hit the adjusting point.

Table 1. Expected Fraction of Casualties Achieved by N Rounds Impacting Within the Target Radius.

Number of Rounds	Target Radius in Meters				
N	25	50	75	100	125
1	.598	.203	.096	.055	.036
2	.838	.366	.183	.108	.070
3	.935	.495	.261	.157	.103
4	.974	.598	.333	.204	.136
5		.679	.397	.248	.166
6		.745	.455	.289	.196
7			.508	.328	.225
8			.555	.365	.253
9				.401	.279
10				.434	.305
11					.330
12					.354
13					
14					
15					
16					

CHAPTER III

REVIEW OF EFFECTIVENESS STUDIES AND COMPUTER MODELS

The review of the literature encompassed articles on damage assessment techniques, artillery studies and methodologies, effectiveness models and numerous papers related to the area of weapons analysis. The diverse number of articles on fire support systems and modeling techniques made the review very comprehensive and very informative. It is intended to discuss some of the pertinent articles and explain in some detail the key simulation models.

Analytical Studies

Breaux and Mohler [6] developed a computational procedure for a class of coverage problems for multiple shots in 1969. The analysis included N rounds of a salvo delivered onto a diffused target where the single round damage function, the distribution of impacts about the aimpoint and the distribution of aimpoints about the target center are elliptical normal. The procedure employed Jacobi polynomials and resulted in better convergence properties of the resulting series. Two new series solutions were presented, one increasing monotonically and the other decreasing monotonically with a summation index. A method of averaging the two solutions was used to accelerate convergence, thereby making the method useful even in extreme cases where numerical difficulties force termination of the series before convergence is reached by either one.

Various studies have examined damage assessment and use several representations for the round lethality function. The most valid comparison was made by the Ballistics Research Laboratory in BRL Report 1544 [21]. The report concerned the method of determining the fractional kill of an area target and compared a poor damage function (the cookie cutter), a good damage function (the exponentional) and the true function. The representation of these functions is shown in Figure 4 where the probability of kill is plotted versus the distance from the burst. The results were that the exponential yielded a much better approximation of the true function than any other.

Bressel [7] evaluated analytically the damage function for a rectangular target from a firing pattern of n rounds by extending to a pattern distribution the method of Grubbs (Operations Research 16). The delivery function and the dispersion of rounds is taken to be non-circular normal, and the damage function law for each round is defined to be non-circular exponential square fall off. This approach is more complex in computations since one must keep track of each round in computing the average chance that a round of the pattern will damage a given target element.

Nadler and Elliott [39] developed optimal sequential aim corrections for attacking a stationary target. The concept involved a battery of weapons being directed one at a time at a stationary target and the impact points had a common circular normal distribution. They derived optimal corrections among all the unbiased corrections that are a linear function of earlier impact point computations. The analysis determined how large the offset distance needed to be to insure a less costly scheme than the strategy of no adjustment to aim.

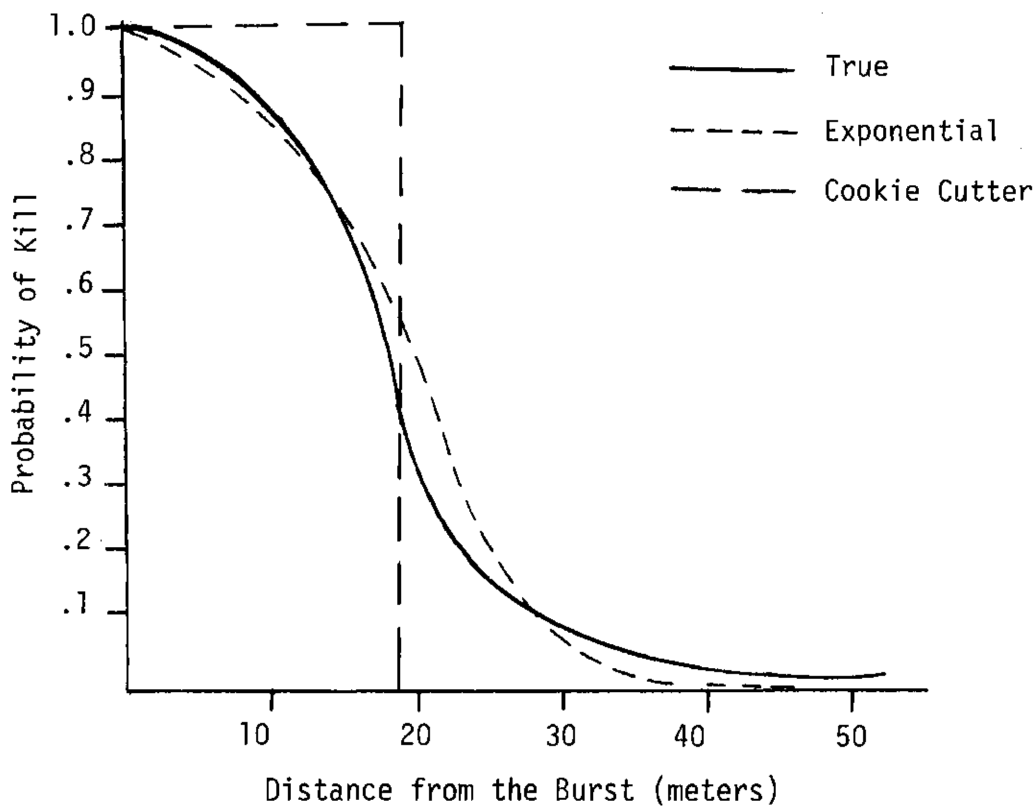


Figure 4. Round Lethality Function.

The importance of Nadler and Elliott's paper is that it showed a bias-correcting scheme is necessary when the bias is large. They determined how large this bias need be to change strategies. To use a bias-correcting scheme it is necessary to introduce a spotting round or an initial volley. At the initiation of firing, it is assumed that the bias and the error are independent and identically distributed normal variables- a state of affairs that can be easily arranged by a calibration weapon or base piece. The two strategies considered assumed that the target is engaged sequentially so that some of the weapons may be saved by observing before a given weapon is committed whether or not the target is destroyed. The distinction between these two strategies arises from the fact that the shoot-adjust-shoot strategy envisages corrections to the aim of the weapon based upon previous impact points relative to the target, whereas the shoot-look-shoot strategy does not permit such readjustment.

The results of the study were tables of probabilities that all weapons in a battery of size n would impact more than a distance R from their target and therefore fail to destroy it. Four bias-correction schemes were used and distances R were varied for $n = 1, 2, \ldots, 8$ weapons per battery. One pertinent conclusion made was that as the battery size increased, the probability of failing to destroy the target greatly decreased as shown in Tables 2 & 3. A comparison of schemes indicated that correcting one-half the distance might be preferred to the optimal scheme of total bias-correction for ease of calculation. A major disadvantage shown was that the impact points of the scheme using one-half bias are so interdependent that analytic expressions for probability of target destruction and expected number of weapons needed for destruction are not available.

Table 2. The Probability That All Weapons in a Battery of Size n Impact More Than a Distance R From Their Target and Therefore Fail to Destroy it for R Equal to 1/2.

Size n	R = 1/2 P_1	P_2	P_3	P_4
1	0.939	0.939	0.939	0.939
2	0.884	0.884	0.864	0.864
3	0.825	0.829	0.786	0.787
4	0.779	0.778	0.722	0.712
5	0.731	0.738	0.656	0.641
6	0.691	0.703	0.600	0.576
7	0.648	0.662	0.539	0.517
8	0.609	0.629	0.484	0.463

Table 3. The Probability That All Weapons in a Battery of Size n Impact More than a Distance R From Their Target and Therefore Fail to Destroy it for R Equal to 1.

Size n	R = 1 P_1	P_2	P_3	P_4
1	0.779	0.779	0.779	0.779
2	0.618	0.618	0.558	0.558
3	0.480	0.497	0.386	0.383
4	0.377	0.403	0.263	0.257
5	0.298	0.333	0.187	0.170
6	0.233	0.284	0.132	0.110
7	0.180	0.244	0.086	0.071
8	0.140	0.210	0.060	0.046

Kimbleton [38] completed an analysis on attrition rates for weapons with Markov-Dependent Fire. The procedure involved using information for each weapon with the result of the current round being conditional on the outcome of the preceding round to obtain the distribution of number of rounds required to defeat a target. The distribution of the time required to destroy the target was derived and the expected value of the Lanchester attrition rate coefficient was obtained.

Witt [56] made a comparison of two target coverage models for a Masters Thesis at the Naval Postgraduate School in 1972. He examined a model for computing the target coverage when multiple rounds are fired at the same aimpoint and compared the results to the second model which used a single shot hit probability for fragment-sensitive targets and then determined the fractional kill which resulted.

The conclusion made by Witt was that there was a significant difference between the results of computations with the model in which it is assumed that the effects of rounds are independent and with the standard salvo-fire model. This difference was more than 300% and usually greater than 200% for the two dimension model in which elliptical normal distributions and representative input data were used. It was concluded that the model which assumes statistical independence of effects of rounds is a very poor approximation to the salvo-fire model.

<u>RAND FORMAC Model</u>

Oman [40] developed for the RAND Corporation a method of evaluating coverage functions, significantly different from existing methods in two respects. First, the method uses a new set of damage functions that are on the one hand empirically realistic, and on the other hand are sufficiently mathematically tractable to allow fairly complicated intervals to be evaluated exactly. Second, the method is implemented on the computer by means of FORMAC, the IBM written symbolic mathematical compiler. The paper is primary written to show an interesting example of how FORMAC may be used when the application of a mathematical approach to an actual real world problem requires cumbersome and involved computations.

Oman viewed the problem of weapons system analysis where it was required to evaluate the probability that a randomly distributed point target (or a fixed target with a distributed mass) will be destroyed by one or more weapons fired simultaneously at it. The probability of destruction can be expressed in terms of a set of multiple integrations whose initial integrands contain distributions relating to the weapons and the targets. The mathematical form of this conditional probability for weapons systems becomes the basis of his application and analysis.

Computer Effectiveness Models

The RAND Model

The RAND Corporation conducted a series of studies concerned with the prediction of damage to a single or multiple targets. Part of RAND's research was on the use of airpower in support of ground operations. One study which is of significance is entitled "A Simplified Weapons Evaluation Model" by Roger Snow and Margaret Ryan, RAND Memorandum RM-5677-1-PR, May 1970 [44].

Several companion studies were done prior to this Memorandum which considered aspects of the target-weapon relationship, target-weapon errors, the coverage problem, and the computer program for a target coverage model. These studies were RM-4566-PR, FAST-VAL: A Theoretical Approach to Some General Target Coverage Problems, March 1966 (FOR OFFICIAL USE ONLY) [45] and RM-4567-PR, FAST-VAL: Target Coverage Model, March 1966 (FOR OFFICIAL USE ONLY) [26].

From the point of view of production computations, however, the target coverage program had two serious limitations: the length of computer time required to make the computations, and the dependence of precision on the size of the integration cell, which in turn depends on the machine capacity available. To alleviate these problems, RAND designed a model that replaced the empirical damage function used in the general model with a simpler and far less time-consuming analytic expression. The results of this work were described in an interim reference report RM 5152-PR which has been withdrawn and replaced by RM-5677-1-PR.

Two restrictions were used by RAND which permitted simplification of the FAST-VAL models mentioned above. The problems were restricted to the case of a Gaussian aiming error distribution and a rectangular target area with uniform distribution of the target elements in the area. As a result it was possible to reduce the coverage computations to two stages, each involving a double integration, in contrast to the three stages required in the original model. The second restriction concerns both the assumed form of the damage function and the ballistic error distribution. It is necessary that (1) the damage function be an analytic function, rather than an empirical function; (2) the ballistic error distribution be one of three types: Gaussian, uniform, or stick type; and (3) the damage function be integrable in a closed form with respect to the ballistic error distribution. Under these restrictions, the coverage computations are reduced to a single stage, involving only one double integration. For some cases, it is possible to reduce the problem to a summation of functions in closed form with no integration necessary.

Two types of damage functions were considered in the RAND model, corresponding to two different types of weapon-target effects: a fragment-sensitive target, or one in which the major damage mechanism is due to fragments; and an impact-sensitive target, or one for which there is a definite geometric figure that must be impacted by the weapon. For the fragment-sensitive target, the empirical function that is usually obtained from a computer program using fragmentation data is replaced by an analytic function, the "Gaussian damage function," fitted through the choice of three parameters. For an impact-sensitive target, it is assumed that the target element is a rectangle, and that there is a fixed probability of damage, given a hit on the element. Under these assumptions the damage function is exact.

The two types of damage function, the aiming and ballistic error distributions, and the basic coverage relation are considered in turn and the expected coverage is expressed as one double integration in terms of the damage function and the various forms of the aiming and ballistic error distribution. For various problems that occur in practice, an explicit expression for the coverage is derived in terms of the pertinent parameters. The set of formulas developed for the coverage function in the RAND model provides an answer to the weapon-target effectiveness problem that corresponds to most of the current weapon delivery systems.

The computer program is approximately 1900 lines in length and the data deck which was used to execute 23 test cases contained over 450 lines. The output of the program is the particular value of expected coverage of the target depending on the parameters considered, such as aiming errors, ballistic errors, target posture and spacing.

The model was modified by the field artillery analysts at Fort Sill and is being used in analyzing weapons effectiveness for battalion type missions over a vast range of target types and postures on the battlefield.

Several limitations exist in the RAND Simplified Weapons Evaluation Model which influence artillery analyses.

(1) The target area used is rectangular in shape.

(2) The damage function is using a cookie cutter method which offers a poor approximation of lethality.

(3) The program length requires a large amount of compilation and execution time as well as machine storage capacity.

(4) The approach in analyzing damage is deterministic in nature.

(5) The ballistic error distributions used are very restrictive in their application.

Legal Mix Studies

The series of studies entitled "Legal Mix" were very pertinent to the artillery analysis research being done. Vector Research, Incorporated, from Ann Arbor, Michigan, conducted the initial studies in 1971 on feasibility of analytically modeling the Legal Mix and Redleg studies of artillery systems [54].

The calculations of artillery effectiveness were performed in two stages:

(1) The computation of expected fraction of losses to the target due to a fire mission, including its cumulative losses from

mission to mission.

(2) Based on the expected losses a specification of whether or not the target is defeated or effectively destroyed and should be removed from the mission list.

The procedures used to calculate the expected fraction of target lost appeared to be based on a series of assumptions adopted for computational simplicity. These assumptions were:

(1) Targets occupy circular areas.

(2) Target elements continually redistribute themselves uniformly throughout the target area.

(3) Ellipticity of location, aim, and delivery errors is not significant.

(4) Given a shell lands in the target circle, the assumption that its entire effects pattern lies in the target circle introduces no significant error.

(5) The linear interpolation scheme used to determine effects as a function of round to round versus occasion to occasion error leads to no significant error.

(6) Reported location errors for the same target to several different firers are independent random errors.

Many of these assumptions appear subject to questions concerning their validity and the error introduced by them. However, based on discussions with the staff at the Army Material and Analysis Agency, it is the understanding that these assumptions were examined for accuracy relative to more detailed, more accurate models available in the literature [Hess, 1968; Guenther and Terragno, 1964]. Within the study contexts to which the Legal Mix model has been applied, the models used were good approximations to more detailed fire effects models.

To determine whether or not a target is damaged sufficiently to be considered defeated, the Legal Mix model compared the computed expected fraction of damage to a threshold value. If the threshold is exceeded, the target is removed permanently from the analysis. Otherwise, the damage is accumulated and the target may be considered in a later mission. It is reasonable to assume that the underlying rational for this procedure is that a unit becomes ineffective when some percentage of its initial number of elements is destroyed.

Although this rationale seems appropriate, it is believed the method of implementing it is not. The comparison of the expected fractional damage to a threshold value suppresses the effect of stochastic variation in the damage producing process in what is believed an unsatisfactory manner. From a population of similar missions with similar expected damage, the fraction of targets destroyed and removed should be equal to the probability that such a mission achieves the actual damage in excess of the threshold. This probability generally will be 0.5 for missions with expected damage approximately equal to the threshold; however, the current logic assumes it is 1.0 for damage just above the threshold and 0.0 for damage just below it.

It is believed that the present logic leads to:

(1) Overstating the success of missions with expected damage above the threshold by a factor of 2.0.

(2) Understanding the success of missions with expected damage just below the threshold.

(3) Reducing the future target population on this erroneous basis, thus possibly under or over rating the number of missions required during later periods of simulated conflict.

Excluding the possibility of cancelling error (the extent of which can be determined by comparative runs of current and stochastic logic), the use of this current logic can lead to two extremely significant errors in overall result presentation:

(1) Very large quantitative errors in rating missions successful.

(2) Differences in the rating of artillery systems and mixes under comparison which are due solely to these errors, rather than to actual performance differences.

It should be noted that the total expected damage figures in the analysis were not subject to errors of the same magnitude. Rather the distribution of losses among targets and comparison of distribution losses with assumed tactical threshold are in question.

There was a table developed to indicate the magnitude of the stochastic effects by consideration of attrition as a Bernoulli process, where

n = the initial number of elements in a target.

p = the expected fraction destroyed used as an estimate of the Bernoulli parameter.

t = the threshold expected-fraction destroyed.

The table depicted (a) the Legal Mix disposition of the target, (b) the probability of reaching the threshold damage using the appropriate cumulative binomial distribution, and (c) the frequency of excess target removals expected by not employing stochastic logic in the Legal Mix model, as a function of n, p, and t. The target was assumed removed if $p \geq t$.

A simple program model would eliminate the probability of excess removals and would use the binomial distribution table look up to compute a close approximation to the probability of success for any single mission or cumulated group of missions. It could use the initial system and total expected fraction of damage as look up parameters and score target defeats in two ways - one for reporting defeats and one for removal of the target from future target streams. For scoring actual expected number of defeats would be reported. For future target removal, targets would be removed on a Monte Carlo or other sampling basis.

There would be some minor developments required to create appropriate rules for two details:

(1) The exact method of computing the probability of success for non-homogeneous targets with threshold dependent on survivors of more than one constituent target type.

(2) The appropriate rule for scoring and sampling from sequential missions at one target.

Measures of Effectiveness. The measures of effectiveness used to compare alternative mixes of fire support systems appear to be damage, resources employed and the number of successful missions completed. These MOE can lead to large biases, either for or against specific weapons systems, when coupled with the different views of the threat seen by the acquisition and fire effects models.

There are three major problem areas associated with the Legal Mix Model. First is the use of one replication of the sensor systems to obtain a single realization of the threat or target. Second, the two views of the threat (acquistion and fire assessment) can lead to large biases since there is no feedback of firing effects to the sensors. Finally, no stochastic effects are included in the model where it is known historically that random effects have a high impact on the results, e.g., the acquisition and firing processes.

Legal Mix V Study

One of the latest artillery studies was a classified report by the U. S. Army Field Artillery School [50]. The basic purpose of the study was to develop five configurations or mixes of artillery units for the direct support artillery battalion. Four alternate organizations were analyzed along with the current configuration and comparisons were made at several levels.

Table 4 indicates the configurations and the composition of each mix. The term "fire unit" refers to the groups of weapons firing at one time, and FDC refers to the Fire Direction Centers controlling fires.

The mixes were compared on the basis of constant effects at the 50,000 personnel casualty level, 2500 Armored Personnel Carrier level, and 5000 truck casualty level. The threshold was used of 60% strength as the cut off for fire unit effectiveness. Falling below this level was considered combat ineffective. The analysis on survivability was made to test the ability of the unit to withstand enemy counteraction and still perform its mission.

The values in Table 5 indicate the numbers of casualties inflicted during the combat period under the mixes and the strategies of war gaming. The abbreviations used represent Batterys (B), Guns (G) and Fire Direction Center (FDC).

Table 4. Composition for the Legal Mix V Study.

Mix	Number Battalions Per Division	Number Batterys Per Battalion	Number M109A1 Per Btry/Battalion	Fire Units	FDC Per Battery
1	3	3	6/18	1	1
2	3	4	8/32	2	1
3	3	4	8/32	2	2
4	3	4	8/32	1	1
5	3	5	6/30	1	1

Table 5. Number of Casualties from Legal Mix V Study.

	Mix 1 3B/6	Mix 2 4B/8G/1 FDC	Mix 3 4B/8G/2 FDC	Mix 4 4B/8G	Mix 5 5B/6G
Personnel	7968	9040	9291	8109	8646
Tanks	235	270	283	242	267
Fld. Arty.	212	258	258	238	248
APC	285	361	389	308	362
Trucks	686	816	836	729	761
Air Def	25	28	29	29	29
Radar	14	18	19	15	18
Military Worth	32,292	43,898	45,181	35,614	41,189

As depicted in Table 5, Mix 3 was superior in performance to all others except for two target types which were equalled by other mixes. The evaluation of these casualties resulted in giving Mix 3 a greater military worth value than the other strategies. Over the duration of 20 combat periods being simulated, Mix 3 was able to average a higher military worth value than all others.

Results

The results indicated that Mix 3 (composed of two fire direction centers per 8-gun battery) was the most effective of the five mixes for one combat period. Mix 3 was rated 1% over Mix 2 and 3% over Mix 5 when relative worth values for 10 and 20 combat periods were used. Mix 3 had the largest percentage of its weapons available at the end of the 24th hour of the combat period.

Legal Mix Conclusions

The best organization for the medium artillery in support of a division is with three battalions of M109A1 Self-Propelled Howitzers each with four batteries and eight guns per battery deployed as two 4-gun units each having its own fire direction center. The next best alternative is the same as above except each battery has only one FDC. The major contribution of the Legal Mix study to the analysis in this paper is implicated by the conclusion that smaller units be employed in the artillery organization. The matter warrants investigation into the number of weapons that should be used in units and the configuration of weapons used in achieving aggregated effectiveness which are the objectives of this research.

The BDM Model

The Braddock, Dunn and McDonald (BDM) Services Company developed a firepower model in 1975 for the purpose of measuring the effectiveness of fire on targets. Porter and Hyams [3] were the designers of the model which was called "The KABOOM Firepower Model." The development of the model was necessary for use in an electronics warfare contract relating to the analysis of using direction finding devices to locate targets for our weapons systems. The basic capabilities of the model and its design specifications will be discussed in detail.

Overview. The indirect fire methodology involves a target acquisition device or system locating an enemy target and estimating the location for the weapon system to engage from a firing position. The fire control system determines aimpoints to be used to destroy the target and allocates rounds among these aimpoints. The probability of kill depends on the accuracy of the target acquisition device, the position of the gun relative to the target, the vulnerability of the target, the accuracy of the gun, and the weapon's aiming strategy. Given the above information, the model uses a computer program that determines the location of the rounds fired and computes the probability of kill.

The effectiveness was measured in terms of P_k, the probability of a kill, a value between zero and one. The model used a damage function in its assessment of lethality of round against a target. The damage function was a relation between the distance that a round falls from a target and the probability of a kill of the target. The standard damage function is described by two input parameters in the model: A and MLR. A is the offset, the distance from the center of the target in which there is an assured kill, should the round fall in this distance. A usually corresponds to the radius of the target or the target radius plus a short distance. MLR is the "Mean Lethal Radius" of the function; MLR is the distance from the edge of the target at which the probability of killing the target is 0.5.

A number of target classes exist that are neither point or area targets and as such it was determined that the damage function of these targets for various weapons could not be characterized as is presented above. For this reason, the damage function in this model was placed in a subroutine entitled PKILL which can be replaced by more exotic and flexible versions when designed.

Another feature of the model is a plotting subroutine which plots for a given Monte Carlo draw, the placement of rounds relative to the target estimation location. This plotting routine can be called at any time and at any draw desired.

The accuracies of the location estimation device and the accuracy of the weapon device are inputs in terms of standard deviations of bivariate normal distributions. This conflicts with some standards which express these accuracies either as 50% Circular Error Probable, 50% Elliptical Error Probable or Range Error Probable and Deflection Error Probable. Appropriate conversions need to be applied when using the model. One special feature of the model is the ability to input aimpoints used by the weapons. The manner in which the aimpoints are inputted is peculiar. The coordinates of each aimpoint is given relative to the origin as the target location estimation and the gun-target line being along the x-axis. This simplifies the calculations by the user a great deal. For example, if the user wanted to fire at two aimpoints 50 meters front and back of the gun-target line, the inputted aimpoints would be (-50, 0) and (50, 0). The model rotates and translates these aimpoints to the actual target estimation along the gun-target line.

The final result is the probability of a kill based on rounds on target and the mean and variance of the probability of a kill for a given strategy in the Monte Carlo simulation.

Model Damage Function. In modeling the combat situation, consider the standard two-dimensional cartesian plane. As an orientation assume that the target acquisition device estimates the target location to be at the origin (0, 0). The probability that the acquisition system or the forward observer with his equipment correctly estimates the target location takes on a bivariate normal distribution, with deviation σ_{XT} and σ_{YT} along the X- and Y-axes, respectively. In general $\sigma_{XT} > \sigma_{YT}$.

The nature of the gun is such that the probability that it will hit its aimpoint takes on a bivariate normal distribution with deviation σ_r along the line-of-sight and σ_d perpendicular to the line-of-sight. In general $\sigma_r > \sigma_d$. In Figure 5, the gun is located at (XG, YG) and is aimed at the origin.

Let (XT, YT) be the actual target location and let (XW, YW) be the point of impact of the shell. The distance between these two points is

$$RD = (XT - XW)^2 + (YT - YW)^2 \,. \tag{3.1}$$

The probability of kill (PK) is a function of RD, and is described as follows:

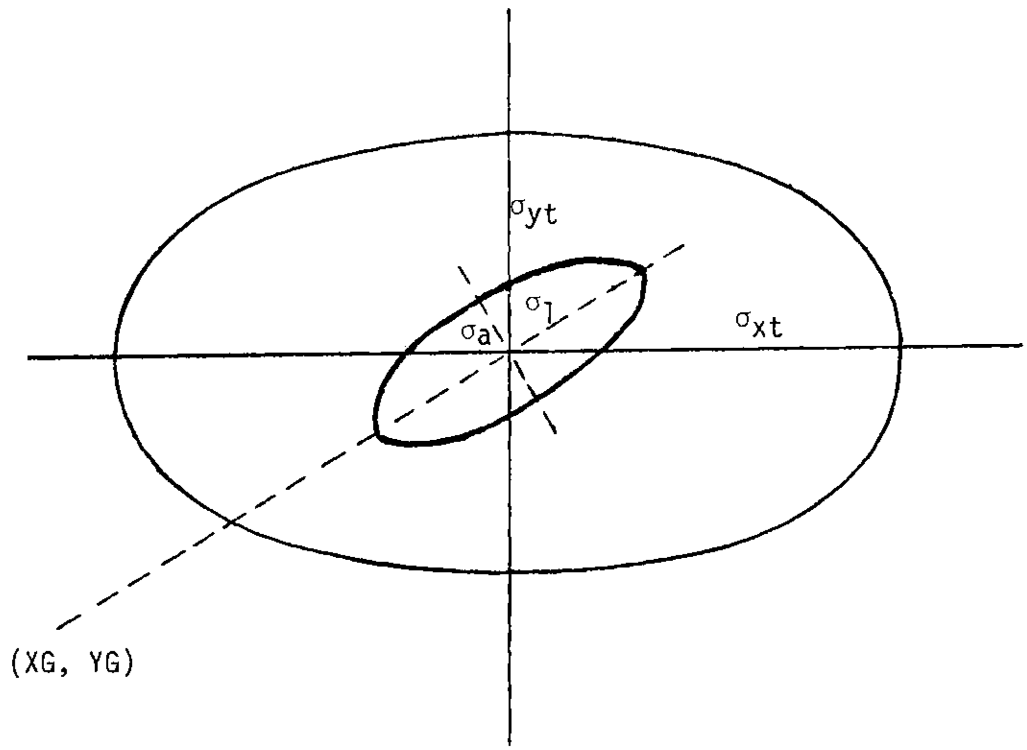


Figure 5. Graphic Representation of Indirect Fire Methodology.

$$PK\ (RD) = 1, \qquad 0 \le RD \le A$$

$$PK\ (RD) = e^{-B(RD-A)}, \qquad A \le RD \le A + MLR$$

$$PK\ (RD = 0.5 - \frac{(RD-A-MLR)}{4}, \qquad A + MLR \le RD \le A + MLR \tag{3.2}$$

$$PK\ (RD) = 0, \qquad RD \ge A + 3\ MLR$$

The definitions of the terms used here are: A is the offset of the damage function, MLR is the mean lethal radius, and B is a constant given by $e^{-B(MLR)} = 0.5$. The graph of the function for PK (RD) is shown in Figure 6.

The simulation of the combat situation is done in a Monte Carlo fashion. The following variables are input: σ_{XT}, σ_{YT}, σ_r, σ_d, XG, YG, A, MLR, and M (the number of rounds). The weapon's aiming strategy must be given by K aimpoints U(i), V(i), i = 1, ..., K, each weighted by a factor L(i), i = 1, ..., K, so that if one chooses his aimpoints from these K aimpoints, he will aim at (U(i), V(i)) $\frac{L(i)}{100\ \Sigma\ L(i)}$ percent of the time. In this analysis the rounds are all equally weighted and correspond to each weapon firing. Each aimpoint therefore corresponds to a weapon.

Determination of Target Location. The target is bivariate-normally distributed around the origin so the mean is zero and the standard deviation is based on the location accuracy. Hence, (XT, YT) is determined as follows:

$$(XT,\ YT) = (0 + \sigma\ N(0,\ 1),\ 0 + \sigma\ N(0,\ 1)) \tag{3.3}$$

Determination of Aimpoints. M is the number of rounds fired per volley. The method of determining the number of rounds to be fired at

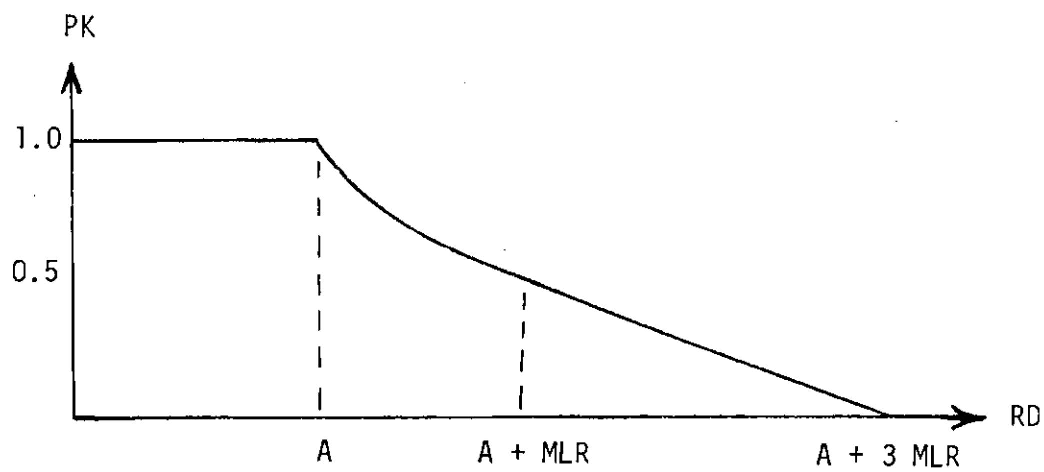


Figure 6. PK as a Function of Distance RD of Round from Target.

each aimpoint is:

$$\{M \cdot L(c)\} / \sum_{i=1}^{K} L(c) \tag{3.4}$$

with integer roundoff as appropriate, i.e., each aimpoint receives its weighted share of all rounds to be fired.

Determination of Target Landing Points. For each i = 1, ..., M, the line-of-sight from the gun location (XG, YG) to the aimpoint XA(i), YA(i) can be characterized by the angle that this line makes with the positive X-axis. The expressions used are:

$$\cos \theta = (XG - XA(i))/R \tag{3.5}$$

$$\sin \theta = (YG - YA(i))/R \tag{3.6}$$

where $R = (XG - XA(i)^2 + (YG - YA(i))^2$.

The coordinate axes for the bivariate normal distribution are then rotated by θ in the counter-clockwise direction, and hence the weapcn location is determined by:

$$XW = XA(i) + \sigma_r \cos \theta \, N(0, 1) - \sigma_d \sin \theta \, N(0, 1) \tag{3.7}$$

$$YW = YA(i) + \sigma_r \sin \theta \, N(0, 1) - \sigma_d \cos \theta \, N(0, 1) \; . \tag{3.8}$$

Computation of RD and PK. As each round impacts near the target, it is evaluated by the damage function to determine how much of the effects inflicted damage. The PK is also determined and both values are computed for each i = 1, ..., M. The total PK, over all M rounds, is given by the expression:

$$PK_T = 1 - \sum_{i=1}^{m} (1-PK_i) \tag{3.9}$$

Monte Carlo Results. The above procedure is run a total of N times to obtain N values of PK_T. These values are then statistically analyzed to give their mean and variance:

$$\mu = \frac{1}{N} \left(\sum_{j=1}^{N} PK_{T_j} \right) \tag{3.10}$$

$$\sigma^2 = \frac{1}{N-1} \sum_{j=1}^{N} (\mu - PK_{T_j})^2 \tag{3.11}$$

Limitations of the BDM Model. The model evaluates the probability of kill based on the rounds on target and can be insensitive to other rounds once the target element has been effectively hit by previous rounds. The results are based on a Monte Carlo method in which a specified number of draws is used in the analysis. Circular target areas and a circular damage function are used in evaluating the system.

Comparison of Three Models

The RAND model (SWEM) has much more detail in evaluating coverage functions and is more general in nature. It uses rectangular target areas and very specific ballistic error distributions. The analysis of damage is deterministic and measures the penetration of the smaller elements within the target area.

The Legal Mix model was limited in its assessment of damage to a Bernoulli process in which either a hit or miss was considered. The possibility of large biases in the analysis was greater due to the technique of no feedback of firing effects. The model did not include the stochastic approach in the analysis which is needed for random effects.

The BDM model used a Monte Carlo process to generate random draws and varied the distribution errors used to describe the location and precision error deviations. The model did not include feedback of effects but could be adapted easily. The capability of using multiple aimpoints to cover the target is very worthwhile and practical. The ability to accumulate the rounds on target and to graphically display them enables the model to evaluate several MOE without difficulty. The program is relatively short in the number of lines to be compiled and is designed to easily execute runs.

CHAPTER IV

DEVELOPMENT OF METHODOLOGY

This chapter on the development of the methodology includes the comparison of weapon configurations, the constraints in the analysis, the measures of effectiveness, the criteria for model selection and the approach used to solve the problem of aggregated effectiveness.

The purpose of evaluating configurations of weapons was to develop an approach for the basic field artillery unit, the battery, to engage targets efficiently and effectively. Previous approaches and studies have examined large numbers of rounds fired by battalion and higher level units. There are numerous combinations which are possible in examining groups of weapons but the design used was to implement realistic groups of guns as they would be employed.

Comparison of Configurations

Consider the requirement that four rounds were needed on a target to achieve the desired level of destruction. The number to fire to achieve the required effects on target would be extracted from an effects table corresponding to the target size. If six rounds were required to be fired to insure that four impacted on target, there are four methods of delivering these six rounds: (1) one volley of six, (2) two volleys of three, (3) three volleys of two or (4) six volleys of one. In considering the unadjusted approach where the rounds are fired at one aimpoint, the volleys would be fired in rapid succession. One would consider firing as many rounds as possible at a time as being the best method; however, this may not be effective if the location is in error or the rounds impact too far from the target to inflict any damage. When smaller groups of rounds are used, more missions may be fired concurrently which increases flexibility in the system. Only when the initial location is exact and the weapons fire accurately, will the unadjusted approach be effective. In most fire missions, it is necessary to adjust fire to correct any bias in the system from initial errors. For this reason the comparison of groups of rounds is essential in determining most feasible approaches.

When the adjusted fire technique is used, the engagement of the target by from two to six rounds at a time may produce better results. In order to compare how well each of the configurations are, there must be a common basis to insure equivalent conditions. A restricted number of total rounds fired would provide a valid basis of comparison. The procedure would be to use combinations of two, three, four and six guns to engage the target. The expected results would be compared using the measures of effectiveness to evaluate any differences in the treatment levels. It was appropriate to choose these levels due to their relation to a common total number of rounds and in the analysis twelve was the total used.

Graphical Analysis

The method of representing the system of locations is to use the two-dimensional coordinate system for the weapons locations and the target. The distance to the target or the range was held constant in the analysis due to its characteristics and to insure comparable effects at all levels. Based on discussions with analysts at Fort Sill and information from the

Joint Munitions Effects Manual analyses the best range for the 155mm howitzer in analyzing effects is 12,000 meters since it is two-thirds the maximum effective range and it is representative of all angles of fall of the projectile at larger ranges. When one location was used to locate all weapons, the location was fixed at a starting point for the range to target. When two locations were required, the second location was approximately 800 meters away since this is the maximum distance elements would be displaced in the BCS design. When a third location was used, the unit was emplaced equidistantly between the first two. This was done to maintain consistency in the total weapons spread in the analysis. The range to the target from the second and third locations was maintained as 12,000 meters since the target was fired in one location, and all locations were engaging the same target.

Constraints in This Analysis

The key assumptions and constraints made in this analysis are based on information commonly used in artillery effectiveness studies and are used to confine the problem to a reasonable level to obtain useful results.

The assumptions relating to effects are as follows:

(1) The concept of lethal area describes the effects of rounds.

(2) All damage effects occur within a circular area.

(3) Impact points are distributed randomly, uniformly and independently within the target circle.

(4) Direct effects of the fire are considered which are achieved by destruction as opposed to indirect effectiveness, i.e.,

suppression of an enemy unit without necessarily causing severe damage or giving other friendly weapons a better chance.

(5) Neutralization of a target is achieved when 30% of the personnel and material are rendered ineffective or 30% of the target area is damaged.

The assumptions relating to target types are:

(1) The type consistency involves personnel, soft equipment like trucks, tents, supplies, etc.

(2) Targets are considered circular in pattern.

(3) The sizes of targets are limited to those considered normal for a battery to engage, up to 100 meters.

(4) Small targets are approximated by a nominal radius, 3 meters, representing a vehicle, command post or radar system.

Since high explosive ammunition is the most commonly used and composes a majority of the military stockpile for this weapon, this analysis only considers high explosive (HE) rounds, M107, in determining the volume of fire required to defeat a target. The fuze type being used is "quick" which impacts on contact with the ground or hard surface. Fuze "variable time" (VT) impacts above the ground and provides more open coverage on the target but the exact dispersion of this type varies so drastically that it is not feasible to show the effects in this analysis.

Most combat engagements in measuring artillery effects are based on a time constraint of five minutes assault time for the target to be engaged. For the 155mm howitzer, the rate of fire is one round per minute for sustained fire. The number of volleys which can be fired in this time period depend on several factors. The training of the gun crews, the fire direction center, and the observer are critical and can cause delays in subsequent volleys as well as increase the volleys during the period. The time constraint can translate to from 2 to 6 volleys of fire being delivered. The basic employment of volleys will be the initial round or rounds fired at time zero and one volley per minute as necessary for the next five minutes. Hence a maximum of six will be used in the analysis. The time for the observer to locate the target and call for fire is not considered but his adjustment time between volleys is a definite element of the time.

Location Accuracy Error

The location probable error accounts for mislocating the target. This error is dependent on the reporting agency, the range, the terrain, the weather, the target size, and the acquisition device. These variables are related and little work has been done to determine their relationship to the location error. This error is most significant and should not be neglected, therefore an approximation should be made.

One approach used by the Field Artillery School was based on an analysis in 1962 by Oklahoma University Research Institute in which the location probable error was related to the target size. The location probable error was included in this analysis with the tabulated firing table errors to yield a total range error and total deflection error. Since the firing table values were nearly constant for all ranges, the total probable error was relatively constant. This notion is very restrictive and is limited in intuitive appeal since it does not lend to variations in the analysis which actually exist in the system.

The approach used for the location probable error included in this research incorporated not only the characteristics error associated with the acquisition method but also a mean point of impact error which is considered when transferring from a previously registered point in the target area to a different location. This latter error is the average value for the weapon system, ammunition and range being evaluated on a transfer type mission. This error is used on the initial volley to assist in getting as close to the target based on the existing firing data corrections. The method of combining the location errors in range and deflection is taken from the Joint Munitions Effects Manual used by the Field Artillery analysts.

The expression which is used to relate the location errors is

$$\sigma \text{ Total Range Error} = \sqrt{(REP_m)^2 + 0.328\ (TLE)^2} \quad \text{or} \tag{4.1}$$

$$\sigma \text{ Total Deflection Error} = \sqrt{(DEP_m)^2 + 0.328\ (TLE)^2} \tag{4.2}$$

depending on which error is being determined. The parameters used in the expression are:

REP_m = the range probable error of the MPI for transferring

DEP_m = the deflection probable error of the MPI for transferring

TLE = the target location error of the acquisition device.

Table 6 indicates the parameter values used for location error for the two acquisition systems, the laser device and the conventional binoculars, in meters.

The results of inputting the parameters in Table 6 into Equations (4.1) and (4.2) are the total range and total deflection errors for each

Table 6. Locational Accuracy Errors for the Acquisition Systems.

System	TLE Range	TLE Deflection	MPI REP_m	MPI DEP_m
Laser	15.0	3.0	89.7	39.8
Conventional	340.0	85.0	89.7	39.8

system which are:

	Range	Deflection
Laser	90.1	39.9
Conventional	214.4	62.9

These values are used to describe one standard deviation in range and in deflection for the elliptical distribution used for locating the target. The model utilizes the error by randomly drawing points from the distribution by the Monte Carlo method and uses this as the actual target location.

Precision Error

The values for the precision error for the 155mm howitzer are extracted from the firing tables for a specific charge and at a specific range. Based on the optimum conditions for the projectile's angle of fall and the best medium range, two-thirds of the maximum range is considered the best for analyzing effects. This range is 12,000 meters and has been used due to the fact that it is representative of the angles of fall of most of the larger ranges. The precision error (associated with round to round) is found to be $PE_{range} = 30$ and $PE_{deflection} = 7$, for 12,000 m. These values must be converted to standard deviations to be used in the model. Based on the definition of the 50% probability ellipse and the relationships between standard deviation and the probable error, the conversion expression is

$$\text{one P. E.} = 0.6745 \text{ std. dev.} \qquad (4.3)$$

and the value of the standard deviation is found by dividing the P. E. by the factor 0.6745. The converted values become $\sigma_r = 44.5$ and $\sigma_d = 10.4$.

Measures of Effectiveness

In the identification of measures of effectiveness, one must consider the following factors:

(1) It should be directly related to the objective of the fire support system and can be analytically tractable or measurable.

(2) Examine the highest level - measures such as the amount of ground lost or gained as a function of time, casualties inflicted, expected outcome of the battle, or number of rounds on target to achieve a level of destruction.

(3) Examine lower levels - accomplishment per weapon operation and continuity of fire support to the supported forces.

(4) Ensure selection that is most closely related to performance measures such as probability of acquiring a target, circular error probable of target location designator, and probability of target destruction.

(5) Costs and other constraining aspects or cost effectiveness techniques to permit direct comparison of alternatives [41].

The measures initially used to determine effectiveness were (1) the number of rounds on target, and (2) the probability of a kill, P_k, based on the damage function. Since rounds on target could be measured, it was essential to use this MOE and compare it to some threshold value which relates to being critical in the analysis. The probability of a kill was based on the damage function but also on where the rounds impacted in order to destroy the target.

Once the initial stages of the analysis were completed, it was determined that P_k may not be indicative of the effectiveness based on the model capabilities. The P_k factor was a function of rounds up to a point and then became insensitive to the number of rounds on target.

When the analysis was changed to include adjusted volleys, the miss distance of the group of rounds fired and the mean radial error of each round were used as measures to indicate closeness to the target. These MOE would indicate how effective the strategy was functioning in converging onto the target.

Determination of Miss Distance

The miss distance is the deviation from the target of the group of rounds fired in each volley and is shown in Appendix A. Each round is compared in coordinates with the target and the difference in the x and y directions is accumulated until the entire volley of rounds is measured. The average of the rounds differences determines a radial miss distance which corresponds to center of the group as a line used to measure total distance from the target as a MOE.

Determination of Mean Radial Error

The mean radial error is defined in Appendix A. The difference in the x and y direction of each round is determined and the radius of location of the round from the target is accumulated for each round in the group being fired. The average of these radial errors becomes the mean radial error and is used as a second measure of effectiveness.

Model Selection

In order to analyze the problem of multi-gun effectiveness and the intra-battery allocation problem peculiar to the BCS, there are several performance criteria which were considered important in this analysis. The criteria are as follows:

(1) A random process generator.

(2) Variation in location accuracy error and gun precision error.

(3) An assessment of rounds on target.

(4) Be capable of multiple aimpoints.

(5) A simple algorithm adaptable to field use.

Based on the comparison of the three key models made at the end of Chapter III, it was determined that the BDM model was best suited for this analysis since its capabilities satisfied most of the criteria which were established.

In the BDM model the design of the damage function was more realistic since it considered not only rounds that impacted within the target radius but also those that landed just outside the radius which produced partial damage. The input parameters could adequately describe the location and precision error distributions involved in the system. The technique for variation in sampling from these distributions was characteristic of the artillery weapons performance. The capability of locating and plotting rounds enabled variations to be easily measured and used in evaluation of MOE. The inclusion of multiple aimpoints enabled a number of rounds to be fired and hence was used as a basis for designing configurations of weapons.

Modification of the Model

The BDM model evaluated the rounds fired in a volley at designated aimpoints at one time and contained no technique for adjusted fire. The resulting response of probability of a kill was not as meaningful as expected in that it was insensitive to the number of rounds on target beyond a specified number. To be able to measure the damage done to a target by a group of rounds it was necessary not only to record where the rounds impacted but also to evaluate the portion of the effects which damaged the target. Since several MOEs were being evaluated, the model was modified to enable the actions necessary to evaluate the MPI and MRE. The mutliple rounds being fired could be adjusted by changing the aimpoints by the desired bias correction to achieve better results. Since subsequent corrections were based on the previous volley, their was dependence created in the system and the method of manual interface was used to make the corrections for subsequent volleys. The shift was made in the opposite direction of the deviation and was rounded to the nearest five meter increments.

An algorithm was employed to determine the MPI of the group of rounds and the MRE for each volley fired. This technique is described in the Appendix A.

Approach for Unadjusted Fire

The procedure for the evaluation of the treatment levels in the unadjusted fire approach was to simulate the firings over a number of runs to determine an expected mean value for the response. The sample size to be used in the Monte Carlo simulation was arbitrary. Based on discussions with analysts at the Artillery School who participated in the Legal Mix Studies it was determined that 15 to 20 observations would be adequate to evaluate how the system performed. A total of 20 runs for each group of weapons was used.

Initially the target size of 100 meter radius was chosen. The aimpoints were varied over the target according to the design specifications for the BCS. It was found that within the precision accuracies of the system the majority of the rounds landed on target. The target size was too large to discriminate between the levels and hence was too insensitive to the variations which existed in the system. The order of magnitude for the target was more than twice the precision standard deviations being used. The decision was made to reduce the target radius to 50 meters and the same sequence of observations was made to determine the achieved rounds on target.

This analysis was based on separate units being independent in their operations in engaging the target. Based on the initial location error being used, it was realized that there was a large bias in the system and this contradicted the hypothesis of independent systems engaging the target. Due to the same inputs being used and the only variation being made was from independent random seeds the systems of using the same configurations at different locations were not independent and therefore no basic difference existed. The deviations in the results were only separate groups of observations from the same distribution. Comparing

2 two-gun units with a four-gun unit made no real difference in this method. By not adjusting the rounds and using the initial aimpoint for all volleys, a large bias was introduced throughout the entire analysis. Since the results indicated that this independence did not exist the manner in which it was proposed, the method was changed to include adjusted volleys being used to engage the target.

Approach for Adjusted Fire

The basis for using adjusted volleys was to eliminate the bias error of the initial volley. By adjusting the rounds closer to the target, the different levels could be evaluated on how well they engaged the target based on the MOE.

The procedure that was used to adjust the rounds was to apply the total correction to move the aimpoint. It was felt that to use other bias schemes was not appropriate since the limit on volleys and time made it more advantageous to converge on the target as soon as possible. Nadler and Elliot [39] used four schemes in evaluating the probability of destroying the target. The results of their analytical method indicated one-half bias corrections were easier to compute and showed better results. However, time was not a factor in their analysis nor was the randomness of the stochastic process. In this paper subsequent corrections were made to adjust each volley unless the rounds were close enough to require no adjustment.

Initially a target radius of 50 meters was used to test the model's performance. It was determined that the size was too large to indicate any difference in the number of rounds on target. The order of magnitude of the target size was comparable to the precision of the system and hence it was decided that no appreciable difference could be obtained. The target size was changed and reduced to a 3 meter radius thereby giving a small adjusting point for the rounds to be measured against. The results are shown in the next chapter.

The explanation of terms used in the computer program and in the analysis is given in Appendix B.

All the input parameters used in the analysis are given in Appendix C.

The computer program for the model is given in Appendix E.

CHAPTER V

DEMONSTRATION OF METHODOLOGY

The demonstration of the methodology used in the analysis will be organized in four sections. The first section involves the unadjusted approach where volleys are fired from different locations in various groups. The second section involves the adjusted fire approach for two target sizes for four treatment levels. The third section includes the analysis of variance (ANOVA) on the adjusted approach and a related analysis of the results. The fourth section involves the determination of rounds on target using one aimpoint for two acquisition methods.

Unadjusted Approach

The initial analysis was made on a 100 meter target radius using the conventional locational error for target location to provide the least accurate system. In the evaluation of effectiveness, the MOE were mean rounds on target and mean PK. The results are shown in Table 7.

When the 50 meter target radius was evaluated using the conventional locational error, the resulting values were smaller for mean rounds on target compared to the 100 meter target. Table 8 has the results of this analysis.

In analyzing the results in Tables 7 and 8 it should be noted that combining two 2-gun units and two 3-gun units and comparing their totals for mean number of rounds to a 4-gun unit and a 6-gun unit respectively does not indicate any appreciable differences. The PK values vary in

Table 7. Results of Unadjusted Approach Using Conventional Location Error on 100 Meter Circular Target for 20 Monte Carlo Draws.

Firing Position	Rounds Per Volley		Mean Rounds on Target	Mean PK
1	2		1.78	0.939
2	2		1.93	1.000
		Total	3.71	
3	4		3.72	0.950
4	3		2.77	0.950
5	3		2.72	0.950
		Total	5.49	
6	6		5.16	0.900

Table 8. Results of Unadjusted Approach Using Conventional Location Error on 50 Meter Circular Target for 20 Monte Carlo Draws.

Firing Position	Rounds Per Volley		Mean Rounds on Target	Mean PK
1	2		1.73	0.938
2	2		1.72	0.986
		Total	3.45	
3	4		3.10	0.949
4	3		2.01	0.925
5	3		2.12	0.882
		Total	4.13	
6	6		4.29	0.947

accordance to how close the rounds impact near the target center.

The hypothesis that there should be a difference between two smaller units and one larger unit was not proven in the unadjusted approach for these larger targets.

Adjusted Approach

The methodology was changed to reflect the capability of adjusting each volley of rounds onto the target. A target radius of 50 meters was used and multiple aimpoints were selected to conform to the weapon configuration on the ground and to cover the target area. The total number of rounds fired in this analysis was twelve (12) and the volleys were limited to a maximum of six.

The results of the different levels are shown in Table 9. A pattern seems to exist in the resulting totals but a large sample size is needed to enable any determination of differences. Sensitivity is not shown in these results which are based only on one series of observations where a total of 12 rounds were fired for each level.

The procedure for applying corrections was to use the total bias scheme which adjusted the aimpoint by the total error observed. These adjustment corrections were made through manual interaction with the computer for each subsequent volley.

The miss distance was utilized as the second MOE in this analysis to determine if the miss distance became a factor in convergence onto the target. The responses are the distances which each group of rounds missed the target and are shown in Table 10 along with the total miss distance and the average miss distance per volley for each configuration. The

Table 9. Rounds on Target for Adjusted Fire on the 50 Meter Circular Target Using Laser Device and Total Bias Correction.

	Rounds Per Volley			
Volley	2	3	4	6
1	1.31	2.20	3.58	5.15
2	1.44	3.00	2.63	4.64
3	2.00	3.00	3.88	
4	1.96	2.15		
5	1.70			
6	2.00			
Total	10.41	10.35	10.09	9.79

Table 10. Miss Distance in Meters for Adjusted Fire on the 50 Meter Circular Target Using Laser Device and Total Bias Correction.

	Rounds Per Volley			
Volley	2	3	4	6
1	99.66	82.72	68.51	85.34
2	22.23	18.32	37.20	33.51
3	19.20	24.46	17.80	
4	19.80	16.80		
5	20.30			
6	16.40			
Total	197.59	142.30	123.51	118.85
Average Per Volley	32.93	35.58	41.17	59.42

definitions of the miss distance is given in Appendix A.

The miss distance was not considered as an effective measure since rounds could be deviate a large distance from the target and still have a small average miss distance. The analysis also indicated that when more volleys were fired, the average miss distance decreased. The introduction of the mean radial error (MRE) as a MOE was made in the analysis to determine if the radial distance to each round from the target would indicate any significant difference in the levels. The definition of the MRE is shown in Appendix A.

A smaller target had to be used to ensure an adjusting point with a smaller order of magnitude as compared to the precision error to achieve any significant comparison. The target radius of 3 meters was established and considered as a small target for adjustment. The same aimpoint was used and was chosen at the center of the target. The MOE used to evaluate effectiveness were rounds on target, the MPI distance and the MRE. The latter MOE was chosen to establish a basis of comparing the error distance of each round as opposed to the error of the group of rounds in MPI. The results of these sequence of four independent series of observations related that variations occurred in a different manner with the smaller target. Each configuration was executed until a total of twelve rounds were fired at each level. The results have been consolidated in Tables 11 through 13. Table 11 contains the total rounds on target for each series of observations at each treatment level. Table 12 contains the average miss distance per volley for the group of rounds for each series. Table 13 contains the average radial error per volley for the series. The actual data which was recorded is in Appendix D.

Table 11. Total Rounds on Target for Adjusted Fire on a 3 Meter Target Using the Laser Device and Total Bias Correction.

	Number of Weapons Per Volley			
Run	2	3	4	6
1	3.26	4.88	5.29	4.45
2	4.90	4.03	6.14	4.73
3	4.98	4.79	5.34	4.27
4	5.00	5.14	5.32	3.48
Mean	4.54	4.71	5.52	4.23

Table 12. Average Miss Distance Per Volley in Meters for Adjusted Fire on a 3 Meter Target Using the Laser Device and Total Bias Correction.

	Number of Weapons Per Volley			
Run	2	3	4	6
1	43.22	37.01	18.63	26.81
2	31.19	28.83	17.98	23.14
3	29.08	26.79	23.87	48.49
4	25.69	23.29	19.53	30.80
Mean	32.30	28.98	20.00	32.31

Table 13. Average Radial Error Per Volley in Meters for Adjusted Fire on a 3 Meter Target Using Laser Device and Total Bias Correction.

	Number of Weapons Per Volley			
Run	2	3	4	6
1	53.41	45.87	37.13	43.90
2	43.28	41.04	32.89	39.76
3	34.82	33.40	25.67	32.79
4	37.27	34.18	32.40	39.81
Mean	42.20	38.62	32.02	39.06

Analysis of Variance

In order to examine the statistical significance of the results in this analysis, the method was to treat the problem as an experimental design with four treatment levels corresponding to each configuration of weapons used in the groups examined. The Analysis of Variance (ANOVA) was conducted on the three measures of effectiveness to evaluate the significance level of the different configurations used in the analysis [29]. The ANOVA results are given in Tables 14, 15, and 16.

Table 14. ANOVA for Rounds on Target.

Source	SS	df	MS	F_o
Levels	3.65	3	1.21	3.45
Error	4.23	12	0.353	
Total	7.88	15		

The F-test for 3 and 12 degrees of freedom for the value 3.45 corresponds to a significance level of 0.05.

Table 15. ANOVA for Average Miss Distance.

Source	SS	df	MS	F_o
Levels	405.25	3	135.08	2.40
Error	675.78	12	56.32	
Total	1081.03	15		

The F-test for 3 and 12 degrees of freedom for the value 2.40 corresponds to a significance level of 0.13.

Table 16. ANOVA for Average Radial Error.

Source	SS	df	MS	F_o
Levels	219.39	3	73.13	1.98
Error	442.10	12	36.84	
Total	661.49	15		

The F-test for 3 and 12 degrees of freedom for the value 1.98 corresponds to a 0.19 significance level.

A comparison of means was made using the Newman-Keuls range test [29] to determine any significant difference between pairs of means for all three MOE.

The results of the range test on the mean rounds on target for the four treatments indicated a significant difference at $\alpha = 0.05$ level only between the means for the 4-gun unit and the 6-gun unit. The range tests for the average miss distance and for the average radial error indicated no significant difference in the means at the 0.05 level as was shown in the ANOVA.

An analysis was made to determine the average number of rounds per volley for the four runs which is shown in Appendix D. The results of this average calculation for each of the cells in the individual runs are given in Table 17.

The average number of rounds on target was expressed as a cumulative number of rounds per volley to enable the determination of the total rounds at each particular volley that impacted on target. The cumulative rounds are shown in Table 18. The average total rounds on target for each configuration are indicated by the last number in each column.

Table 17. Average Number of Rounds on Target Per Volley for Four Each Observations.

	Number of Weapons			
Volley	2	3	4	6
1	0.252	0.570	1.115	1.952
2	0.785	1.290	2.140	2.280
3	0.558	1.625	2.268	
4	1.032	1.225		
5	0.825			
6	1.090			

Table 18. Cumulative Rounds on Target Per Volley for Adjusted Fire on a 3 Meter Target Using Laser Device and Total Bias Correction.

	Number of Weapons Per Volley			
Volley	2	3	4	6
1	0.252	0.570	1.115	1.952
2	1.037	1.860	3.255	4.232
3	1.595	3.485	5.523	
4	2.627	4.715		
5	3.452			
6	4.542			

Discussion of Results

The observed results of the analysis indicated that a four-gun unit achieved more rounds on target than the other three configurations which were evaluated. The miss distance of the group of rounds from the target and the average radial error of the rounds were minimized in the four-gun configuration. The results were dependent on the constraints used in the analysis and the design of the experiment.

Several factors had an influence in the analysis which should be considered. First, the initial location error difference established the original bias and had an initial influence on the values in that treatment level. Second, the correction procedure scheme was critical in the functioning of the system. The total bias error was corrected in each volley and other correction schemes may provide different results. Third, the number of corrections used in each treatment level drives the process and the more rounds that are fired the closer the group is moved towards the target.

A Related Analysis

A related analysis was made using the results obtained in Table 18. Based on a total of 12 rounds being fired from the groups of weapons which are independent in their functioning and method of attacking the target, one may infer what the expected results would be for designated configurations from the cumulative average number of rounds per volley.

There are several alternatives for a six-gun unit to fire on a target a total of 12 rounds in two volleys. Table 19 indicates four mixes and three different positions.

Table 19. Alternate Groupings

	Firing Positions		
Mix	A	B	C
1	2	2	2
2	3	3	
3	4	2	
4	6		

Considering the assumption of independence of operations of the groups of weapons and using the expected number of mean rounds on target per configuration, the following table is formed by extracting the respective values from Table 18 and accumulating the total. The resulting values are shown in Table 20 for the alternatives.

Table 20. Total of Expected Rounds on Target

Mix	A	B	C	Total
1	1.037	1.037	1.037	3.111
2	1.860	1.860		3.720
3	3.255	1.037		4.292
4	4.232			4.232

The interpretation of these results indicate that based on 12 rounds being fired the optimum mix is number 3 with two volleys each from a four-gun unit and a two-gun unit resulting in 4.292 rounds on target. Comparing this result with three volleys of 4 guns, as was done in the analysis, indicates that 5.523 rounds impacted on target which is better than using the four-gun and two-gun units. In evaluating the firing of four volleys of three-guns, the analysis indicated 4.710 rounds were on target which also was better than the mix number 3. However, the difference in this reasoning which has a definite influence is that more times was required to fire these last two cases since more volleys were used to delivery the rounds.

Determination of Rounds on Target

The demonstration of the number of rounds expected to impact on target was shown by executing a number of Monte Carlo runs and evaluating the mean of the samples. A radius of target of 50 meters was used and both of the locational errors associated with the laser and conventional devices were used to determine location accuracy of the target. A sample size of 20 runs was used for each number of weapons firing which was increased from one up to twelve. The rounds were aimed at the origin or target center to provide an evaluation of the variation which can be expected using one aimpoint. The tabulated results are given in Appendix D.

The interpretation of these results is that to achieve a number of rounds on target to inflict a specified level of damage you would have to fire the number indicated. If three rounds were needed on a 50 meter target to achieve 30% casualties, the Monte Carlo method indicated that on the average firing four rounds would achieve the desired rounds using a conventional location error. Comparing this result to the current effects tables being used which utilize a deterministic approach to achieve damage indicates a large discrepancy in the number of rounds required. An unclassified table based on the JMEM analysis, Table 21, indicates that for

30% casualties in a 50 meter target radius seventeen (17) battery volleys are required or 102 rounds being fired. The table which is used from the effects tables is based on observer adjusted fire on area targets. The fact that such as large number of rounds is required when other analyses indicate much fewer actually are needed is enough reason to question the validity of the current effects tables.

Table 21. Battery Volleys For 155mm Howitzer.

	% Casualties			
	0.30		0.20	
	Fuze Type		Fuze Type	
Target Radius	PD	VT	PD	VT
50	17	8	12	5
100	22	10	17	7
150	28	13	27	10
200	P	24	P	16
250	P	P	P	

Note 1. P indicates prohibitive firing over 30 battery volleys.

Note 2. The table is generated under observer adjusted fire on area targets. The source of the data is Graphical Munitions Effects Table (GMET).

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Limitations of the Research

This research has been limited by the initial assumptions of fixed effects, one weapon type (155mm), indirect fire methodology, fixed targets, a time constraint, a fixed number of rounds and a limited number of volleys. The simulation technique involved the Monte Carlo method for the random process generation. The methodology was designed to examine an increasing number of weapons to determine the aggregated performance level of the unit. The configurations 2, 3, 4 and 6 guns were employed and the resultant effects were compared by the analysis of variance methods and the range tests.

Conclusions

The firing of four guns at a time in subsequent volleys was determined to be more effective by impacting more rounds closer to the target center and by having the least average radial error and the least miss distance of all configurations. The analysis verified the Legal Mix V Study result that the most effective mix of weapons was the two four-gun units operating independently. The results of the ANOVA indicated that the number of rounds on target was significant at the 0.05 level for the different configurations. The ANOVA indicated the average miss distance per volley was significant at the 0.13 level and the average radial error per volley was significant at the 0.19 level for the different levels.

The simulation model used in the analysis is an excellent vehicle for analyzing artillery systems and the alternative adjustment techniques under the limited rounds constraint.

The model was able to yield the expected number of rounds on target when given the number to be fired under the designated input parameters. This information can be utilized to generate effects tables based on the Monte Carlo method. The demonstrated results drastically differed with the munitions effects tables currently being used.

Recommendations

Several recommendations arose for future analysis in the course of this research. Since this research only established a basis of selection of weapons to be used, it was limited by the experimental design and the assumptions for this analysis.

A revised algorithm should be developed for the Battery Computer System as an internal subroutine based on the results of the model used in this research. The computer program was modified to conform to the requirements of this analysis. Further revision is necessary to include adjustment techniques which can be executed within the computer to assist in the analysis.

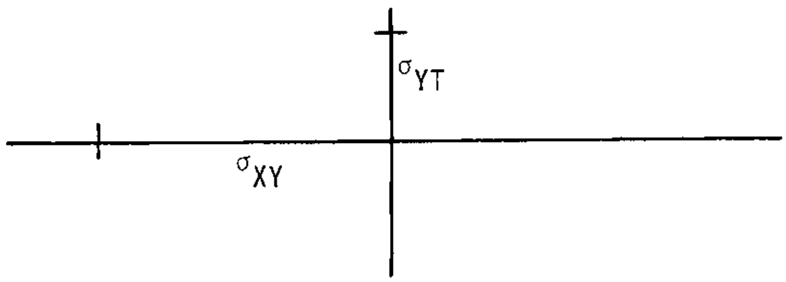
The analysis should be extended to other target types and to various target persistency. Only circular targets were considered in the analysis but other configurations such as elliptical and rectangular targets should be investigated. The limited number of volleys was established based on a five-minute assault criteria. Targets may be exposed or vulnerable for longer periods of time and this constraint should be varied to evaluate long range effects on more persistant targets.

The damage effects tables should be revised and updated to reflect new technology. The use of the Monte Carlo method of generating outcomes yields an excellent approximation of the random process associated with impacting rounds. The expected number of rounds on target can be more closely approximated using this stochastic approach and tabulated effects should be generated using a more efficient system.

APPENDIX A

GLOSSARY

Accuracy - This term represents the measure of how close the estimated location of the target is to the true location. The observer uses either a laser device or binoculars to locate targets and bursts of rounds. How exact he is in determining these locations has been measured and is described by an elliptical pattern. The standard deviations of these location methods are expressed as σ_{XT} and σ_{YT}, along the semi-major and semi-minor axes respectively.



Circular Error Probable - Circular error probable (CEP) is the measure of deviation from the mean and represents the radius of a circle which will contain 50% of all the observations when the mean of the distribution pattern represents the target center. A two-CEP circle is twice the radius of a one CEP circle and includes 94% of the volleys fired. Four CEPs contains essentially all observations fired at the center of the circle.

The relationship of circular error probable to standard deviations can be easily described. Certain conditions have to exist for the circular error probable to have any meaning. These conditions are independent random samples must be used, a bivariate normal distribution must exist.

and a large sample size taken. The firing tables for the artillery weapons are based on these contingencies.

The CEP is often used to describe the accuracy of a system and is directly related to the standard deviation which exists in analyzing populations. The CEP equals 1.1774 times the population standard deviation for the existing σ_e and for the northing σ_n when σ_e equals σ_n. When σ_e does not equal σ_n the CEP is called the equivalent CEP and equals 0.5887 $(\sigma_e + \sigma_n)$.

<u>Mean Point of Impact</u> - The exact center of a large group of rounds is called the mean point of impact (MPI). The center is determined by the intersection of two lines, one perpendicular to the line of fire that divides the points of impact into two equal groups and one parallel to the line of fire that divides the rounds into two equal groups. These lines represent the mean range and mean deflection of the rounds fired. The distance of this point from the adjusting point of the observer is the MPI distance. See Figure A-1.

<u>Mean Radial Error (MRE)</u> - The mean radial error is taken from the circular normal distribution and is the expected value of the radial error P. Various relationships exist between the standard deviation, σ, the circular probable error and the MRE.

$$(\text{M.R.E.}) = 1.2533\ \sigma = 1.0645\ (\text{C.P.E.})$$

In this analysis the radial error for each impact point to the target is measured and the mean of the group of rounds is determined as a measure of closeness to the target. See Figure A-2.

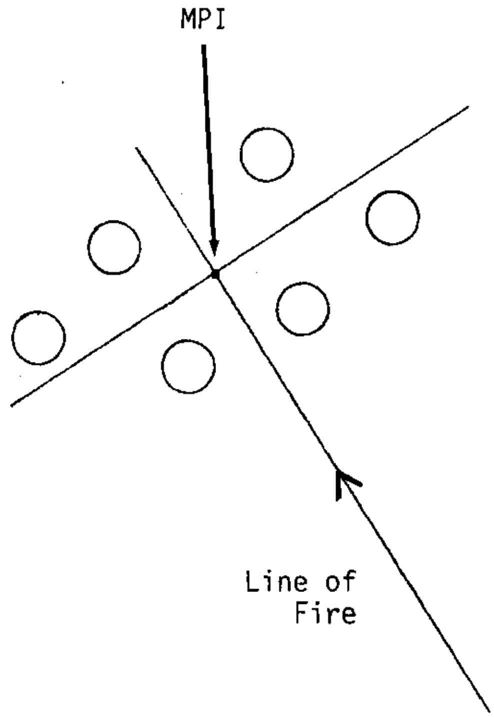
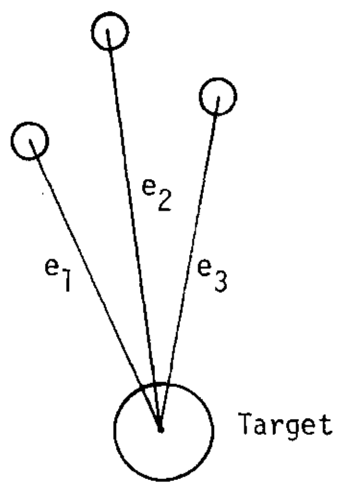


Figure A-1. Mean Point of Impact (MPI)
of a Group of Six Rounds.



$$MRE = \frac{e_1 + e_2 + e_3}{3}$$

Figure A-2. Average Radial Error (MRE) for a Volley of Three Rounds.

Miss Distance - The measure of the center of the impact of a round or group of rounds from the target is being called the miss distance. When a group of rounds are measured, the distance of each round from the target is accumulated in the x and y direction and the average is obtained in two dimensions. This describes the centroid of the plane formed by the group of rounds and is translated to a radial distance for the group. See Figure A-3.

Precision - The measure of how the weapon system performs with respect to effects on the ground is called precision. The dispersion of the rounds on the ground follow a bivariate normal distribution and conform to an elliptical pattern. The method of describing the precision of the artillery system is to specify the standard deviations in range and in deflection (laterally) for the particular range to the target. These standard deviations are converted from probable errors in the artillery firing tables, AM-155.

Probable Error - One probable error is the distance from the mean point of impact to a line which describes the error which is exceeded as often as it is not exceeded. Probable error is also manifested by the rounds which fell short of the mean point of impact.

In a normal burst pattern the number of rounds short of the MPI will be the same as the number of rounds over the MPI. Probable error will be the same in both cases.

It is a coincidence of nature for any normal distribution such as the artillery dispersion pattern a distance of four probable errors on either side of the mean point of impact will include virtually all the rounds in the pattern.

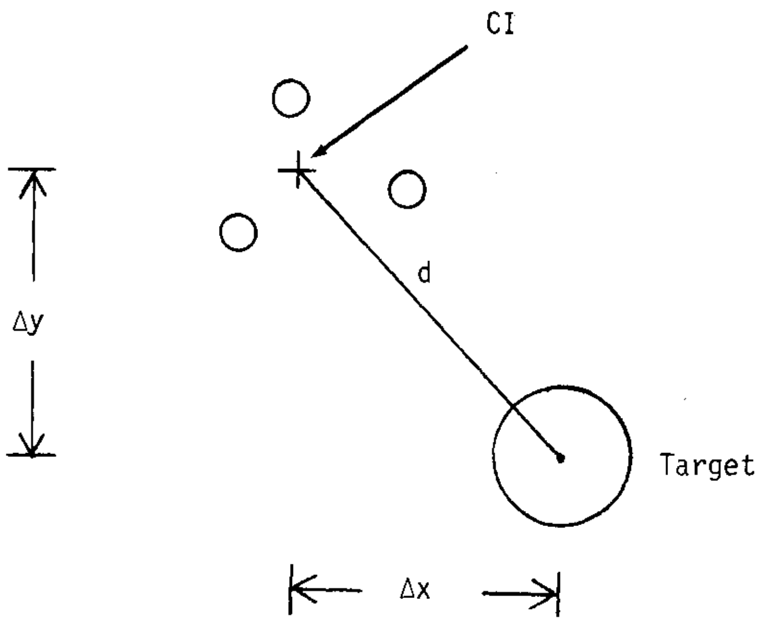


Figure A-3. Average Miss Distance, d, of a Volley of 3 Rounds. CI is the center of impact and Δx and Δy are differences of CI from the traget center.

The total pattern of a large number of bursts is roughly elliptical. However since four P. E. on either side of the mean point of impact will encompass all rounds, a rectangle normally is drawn to include the full distribution of the rounds.

The existing conditions for Probable Error to having meaning are independent (random) samples, a normal distribution and large sample size.

APPENDIX B

EXPLANATION OF TERMS

A	Offset of damage function, radius of target
B	Constant, internally computed for the damage function
CEP	Circular error probable
DEP	Deflection error probable
IMM	Random number seed
L	Weighting factor assigned to aimpoints
M	Number of rounds fired
MEAN	Mean of a number of Monte Carlo draws
MLR	Mean lethal radius of damage of rounds
MPI	Mean point of impact of a group of rounds
N	Number of Monte Carlo draws
ORIENT	Orientation of semi-major axis of locational accuracy ellipse measured counterclockwise from x-axis, in degrees
PC	Probability that map analysis is correct
PE	Probable error
PK	Probability of a kill
PKILL	Subroutine used to determine PK
PLOT	Subroutine used to plot location of rounds relative to estimated target location
RANF	Fortran internal function for random numbers
REP	Range error probable
RD	Radial distance between point of impact of rounds and target location
RNORM	Subroutine which draws a random number from uniform distribution between (0, 1)
ROUND	Number of rounds that impact within the target radius
SIG	Standard deviation
SIGL	Standard deviation for precision error in range

SIGA Standard deviation for precision error in deflection or perpendicular to line-of-sight

SIGXT Standard deviation along semi-major axis of locational accuracy ellipse, in meters

SIGYT Standard deviation along semi-minor axis of locational accuracy ellipse, in meters

SUM Sum of rounds used for determining the mean

SUMSQ Sum of squares of variable for determining variance of rounds

TLE Total location error used to determine accuracy to target

U X-axis coordinate difference for aimpoint

V Y-axis coordinate difference for aimpoint

VAR Variance of the sample being evaluated

X X-axis coordinate of target location estimation

XA X-axis translated distance to aimpoint

XG X-axis location of the weapons system

XT X-axis coordinate of actual target location

XW X-axis coordinate of impact point of round

Y Y-axis coorindate of target location estimation

YA Y-axis translated distance to the aimpoint from the weapon

YG Y-axis coordinate of the weapons system

YT Y-axis coordinate of actual target location

YW Y-axis coordinate of impact point of round

APPENDIX C

INPUT PARAMETERS

This appendix contains the input parameters used in the computer program to generate the observations used in the analysis.

I. Unadjusted Approach with Conventional Location Accuracy Against a 100 Meter Radius Target.

A. Input Parameters:

A = 100.0	XG = 0.0
MLR = 25.0	YG = 0.0
PC = 0.80	M = 2
SIGXT = 214.40	N = 20
SIGYT = 62.90	X = 11619.0
SIGL = 44.50	Y = 3000.0
SIGA = 10.40	ISEED = 123456789

B. Aimpoint Data: L = 1 for all aimpoints.

For M = 2:	U = 0.0	V = -50.0
	U = -50.0	V = 0.0
For M = 3:	U = 0.0	V = 50.0
	U = 43.3	V = -25.0
	U = -43.3	V = -25.0
For M = 4:	U = 0.0	V = 50.0
	U = 50.0	V = 0.0
	U = 0.0	V = -50.0
	U = -50.0	V = 0.0
For M = 6:	U = 0.0	V = 50.0
	U = 43.3	V = 25.0
	U = 43.3	V = -25.0
	U = 0.0	V = -50.0
	U = -43.3	V = -25.0
	U = -43.3	V = 25.0

II. Unadjusted Approach with Conventional Location Accuracy Against a 50 Meter Radius Target.

A. Input Parameters: All were the same as the 100 meter target except that A = 50.0.

B. Aimpoint Data: L = 1 for all aimpoints and values for U and V were reduced to one-half the amount of the 100 meter target.

For M = 2:	U = 0.0	V = -25.0
	U = -25.0	V = 0.0
For M = 3:	U = 0.0	V = 25.0
	U = 21.6	V = -12.5
	U = -21.6	V = -12.5
For M = 4:	U = 0.0	V = 25.0
	U = 25.0	V = 0.0
	U = 0.0	V = -25.0
	U = -25.0	V = 0.0
For M = 6:	U = 0.0	V = 25.0
	U = 21.6	V = 12.5
	U = 21.6	V = -12.5
	U = 0.0	V = -25.0
	U = -21.6	V = -12.5
	U = -21.6	V = 12.5

C. Firing Positions: When a second location was used for the weapons unit, XG = 237.0 and YG = 800.0 while the target location was the same. When a third location was used, XG = 119.0 and YG = 343.0. These values were chosen to maintain a constant range to target of 12,000 meters and to equispace the firing locations.

III. Adjusted Approach with Laser Location Accuracy Against a 50 Meter Radius Target.

A. Input Parameters:

A = 50.0	XG = 1000.0
MLR = 25.0	YG = 1000.0
PC = 0.80	M = 2
SIGXT = 90.10	N = 1
SIGYT = 39.90	X = 12619.0
SIGL = 44.50	Y = 4000.0
SIGA = 10.40	ISEED (See values in D)

B. Aimpoint Data: All aimpoints used were the same as those used in the Unadjusted Approach for a 50 meter radius as shown in II.

C. Subsequent Volleys: The only changes made to the input besides the adjustment corrections for the aimpoint were to use PC = 0.95, SIGXT = 15.0 and SIGYT = 3.0. Since the initial volley determined the position of the target to the rounds, the MPI location error was taken out of the TLE and shifts became more accurate.

D. Input Random Number Seeds, ISEED:

Volley	M = 2	Volley	M = 3
1	374189654	1	562341895
2	524198735	2	648275391
3	394582717	3	824176931
4	356824195	4	628719537
5	435186297		
6	347628519		

Volley	M = 4	Volley	M = 6
1	374189645	1	386475971
2	524179513	2	348921765
3	347891465		

IV. Adjusted Approach with Laser Location Accuracy Against a 3 Meter Radius Target.

A. Input Parameters: All input values used were the same as in III with the 50 meter target except A = 3.0 was used.

B. Aimpoint Data: All aimpoints were the center of the target and values used were L = 1, U = 0.0 and V = 0.0 for the initial volleys and the corresponding corrections for the subsequent volleys.

C. Subsequent Volleys: Besides the adjustments used to change the aimpoints, the three input parameters which were changed were PC = 0.95, SIGXT = 15.0 and SIGYT = 3.0.

D. Input Random Number Seeds, ISEED:

TEST SET 1

Volley	M = 2	Volley	M = 3
1	569872317	1	628719537
2	394582717	2	658413975
3	298465173	3	648275391
4	589123794	4	257468193
5	814356925		
6	621984731		

Volley	M = 4	Volley	M = 6
1	347819645	1	386475971
2	984251367	2	348921765
3	871625739		

TEST SET 2

Volley	M = 2
1	456829713
2	236587413
3	417258637
4	135428761
5	356123897
6	923841762

Volley	M = 3
1	863514796
2	751398547
3	369542187
4	786312498

Volley	M = 4
1	786234195
2	769813427
3	863247189

Volley	M = 6
1	786321593
2	743692157

TEST SET 3

Volley	M = 2
1	618719234
2	541369752
3	258694173
4	746521398
5	913468527
6	432581694

Volley	M = 3
1	243176895
2	956813745
3	784239516
4	574139856

Volley	M = 4
1	378421695
2	258713695
3	731584693

Volley	M = 6
1	156834793
2	763184293

TEST SET 4

Volley	M = 2	Volley	M = 3
1	568723915	1	756321894
2	325741689	2	138654792
3	432589615	3	921367458
4	352687193	4	543217836
5	468319715		
6	761238549		

Volley	M = 4	Volley	M = 6
1	321458739	1	246987317
2	435971385	2	731846592
3	732569815		

V. Determination of Rounds on Target using Conventional Location Accuracy Against a 50 Meter Radius of Target for 20 Monte Carlo Draws.

A. Input Parameters

A = 50.0	XG = 1000.0
MLR = 25.00	YG = 1000.0
PC = 0.90	M = 1, 2, ... 12
SIGXT = 214.40	N = 20
SIGYT = 62.90	X = 12619.0
SIGL = 44.50	Y = 4000.0
SIGA = 10.40	ISEED = 347628519

B. Aimpoint Data

L = 1 U = 0.0 V = 0.0

VI. Determination of Rounds on Target Using Laser Location Accuracy Against a 50 Meter Radius Target for 20 Monte Carlo Draws.

A. Input Parameters

A = 50.00	SIGXT = 90.10
MLR = 25.00	SIGYT = 10.40
PC = 0.90	SIGL = 44.50

SIGA = 10.40	N = 20
XG = 1000.0	X = 12619.0
YG = 1000.0	Y = 4000.0
M = 1, 2, ..., 12	ISEED = 347628519

B. Aimpoint Data

L = 1 U = 0.0 V = 0.0

NOTE: A value of L = -1 after the last aimpoint values for L, U, and V for a volley indicates the last aimpoint to be used for a particular run.

APPENDIX D

OUTPUT DATA

This appendix contains the output data which was generated by the computer program. There are four types of tables represented for rounds on traget, average miss distance per volley, average radial error per volley and base level rounds on target.

Table D-1. Rounds on Target for Adjusted Fire on the 3 Meter Radius Target Using Laser Device and Total Bias Correction.

TEST SET 1

	Number of Rounds Per Volley			
Volley	2	3	4	6
1	0.19	0.14	0.66	2.20
2	1.27	1.03	2.60	2.25
3	0.55	1.73	2.03	
4	0.52	1.98		
5	0.31			
6	0.42			
Total	3.26	4.88	5.29	4.45

TEST SET 2

	Number of Rounds Per Volley			
Volley	2	3	4	6
1	0.69	1.07	1.30	2.75
2	0.46	0.86	2.70	1.98
3	0.60	1.16	2.14	
4	1.19	0.94		
5	0.56			
6	1.40			
Total	4.90	4.03	6.14	4.73

Table D-1. (Continued)

TEST SET 3

Volley	Number of Rounds Per Volley 2	3	4	6
1	0.00	0.61	1.28	1.43
2	0.65	1.32	1.81	2.84
3	0.54	1.78	2.25	
4	1.36	1.08		
5	1.31			
6	1.15			
Total	4.98	4.79	5.34	4.27

TEST SET 4

Volley	Number of Rounds Per Volley 2	3	4	6
1	0.13	0.46	1.22	1.43
2	0.76	1.95	1.45	2.05
3	0.54	1.83	2.65	
4	1.06	0.90		
5	1.12			
6	1.39			
Total	5.00	5.14	5.32	3.48
Average Total	5.54	4.71	5.523	4.232

Table D-2. Miss Distances Per Volley in Meters for Adjusted Fire on 3 Meter Target Using Laser Device and Total Bias Correction.

TEST SET 1

Volley	2	3	4	6
1	76.25	95.32	26.15	30.21
2	28,78	31.10	16.32	23.41
3	49.46	11.20	13.41	
4	31.87	10.41		
5	62.56			
6	10.42			
Total	259.34	148.03	55.88	53.62
Average	43.22	37.01	18.63	26.81

TEST SET 2

Volley	2	3	4	6
1	15.09	18.19	35.20	30.21
2	51.67	51.61	8.16	16.07
3	34.77	23.19	10.49	
4	23.40	22.32		
5	46.05			
6	16.14			
Total	187.12	115.31	53.85	46.28
Average	31.10	28.83	17.98	23.14

Table D-2. (Continued)

TEST SET 3

Volley	2	3	4	6
1	77.68	54.88	45.98	56.18
2	37.95	20.30	16.05	40.80
3	21.46	16.34	9.58	
5	10.29			
6	12.97			
Total	174.52	107.16	71.61	96.98
Average	29.08	26.79	23.87	48.49

TEST SET 4

Volley	2	3	4	6
1	42.66	50.78	26.18	47.95
2	39.16	11.89	20.33	13.64
3	24.46	17.69	12.06	
4	16.72	12.79		
5	15.20			
6	15.94			
Total	154.14	93.15	58.57	61.59
Average	25.69	23.29	19.53	30.80

Table D-3. Average Radial Error Per Volley for Adjusted Fire on the 3 Meter Radius Target Using the Laser Device and Total Bias Correction.

TEST SET 1

	MRE			
Volley	2	3	4	6
1	76.39	95.32	61.49	43.05
2	19.76	45.61	19.87	44.74
3	52.55	24.24	30.04	
4	51.84	18.31		
5	62.78			
6	57.14			
Total	320.46	183.48	111.40	87.79
Average	53.41	45.87	37.13	43.90

TEST SET 2

	MRE			
Volley	2	3	4	6
1	43.55	42.94	46.75	33.91
2	55.07	54.53	17.99	45.61
3	48.96	39.95	33.92	
4	44.85	26.74		
5	50.24			
6	17.02			
Total	259.69	164.16	98.66	79.52
Average	43.28	41.04	32.89	39.76

Table D-3. (Continued)

TEST SET 3

	MRE			
Volley	2	3	4	6
1	59.18	57.75	22.22	48.15
2	45.70	36.00	36.49	17.42
3	33.36	22.92	18.32	
4	26.90	16.93		
5	18.31			
6	25.44			
Total	208.89	133.60	77.03	65.57
Average	34.82	33.40	25.67	32.79

TEST SET 4

	MRE			
Volley	2	3	4	6
1	56.19	71.27	47.47	43.45
2	39.86	19.54	32.04	36.16
3	51.43	27.90	17.69	
4	36.01	18.01		
5	23.97			
6	16.17			
Total	223.63	136.72	97.20	79.61
Average	37.27	34.18	32.4	39.81

Table D-4. Base Level Rounds on Target Without Adjustments Using Conventional Location Error on 50 Meter Radius Target for N = 20 Monte Carlo Draws.

Number of Guns, M	Mean	Standard Deviation (STD)	STD/$\sqrt{N}$
1	.7535	.3565	.7164
2	1.7313	.4692	.1049
3	1.8356	.9145	.2045
4	3.1037	.9563	.2138
5	3.4897	1.1447	.2560
6	4.3825	1.2213	.2731
7	5.4664	1.2892	.2883
8	5.533	1.732	.3872
9	6.9839	1.5784	.3529
10	7.0220	1.8702	.4182
11	7.3947	2.9369	.6567
12	8.6313	3.2037	.7164

Table D-5. Base Level Rounds on Target Without Adjustments Using Laser Location Error on 50 Meter Radius Target for N = 20 Monte Carlo Draws.

Number of Guns, M	Mean	Standard Deviation (STD)	STD/ $\sqrt{N}$
1	0.8442	.3099	.0693
2	1.8077	.4374	.0978
3	2.4861	.7363	.1646
4	3.4961	.4782	.1069
5	4.1334	1.046	.2339
6	5.0788	1.3107	.2931
7	6.2549	.5392	.1206
8	6.3493	1.6298	.3644
9	8.1491	.7014	.1568
10	8.6844	.7342	.1642
11	9.2838	1.609	.3598
12	10.0252	2.4064	.5381

APPENDIX E

COMPUTER PROGRAM

This appendix contains a general flowchart diagram for the computer program and the complete FORTRAN program listing to execute the analysis to include subroutines and functions.

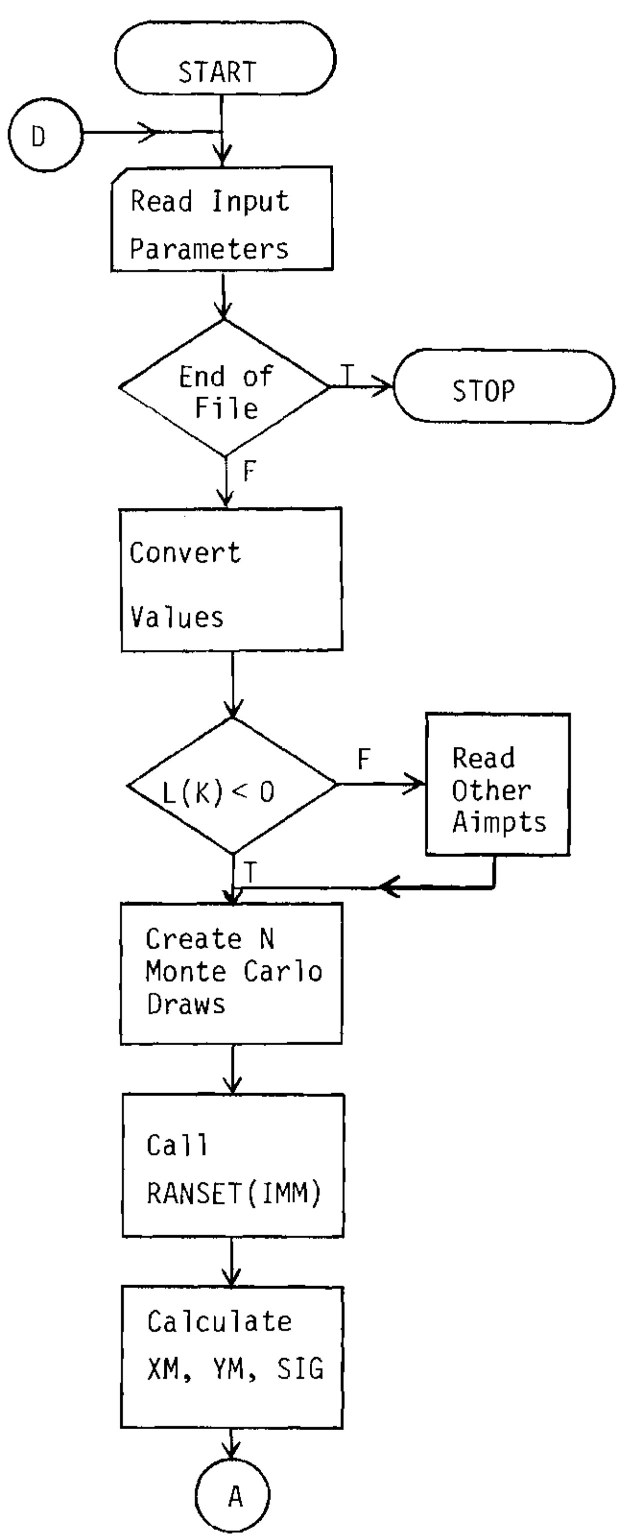


Figure E-1. Flowchart for Program.

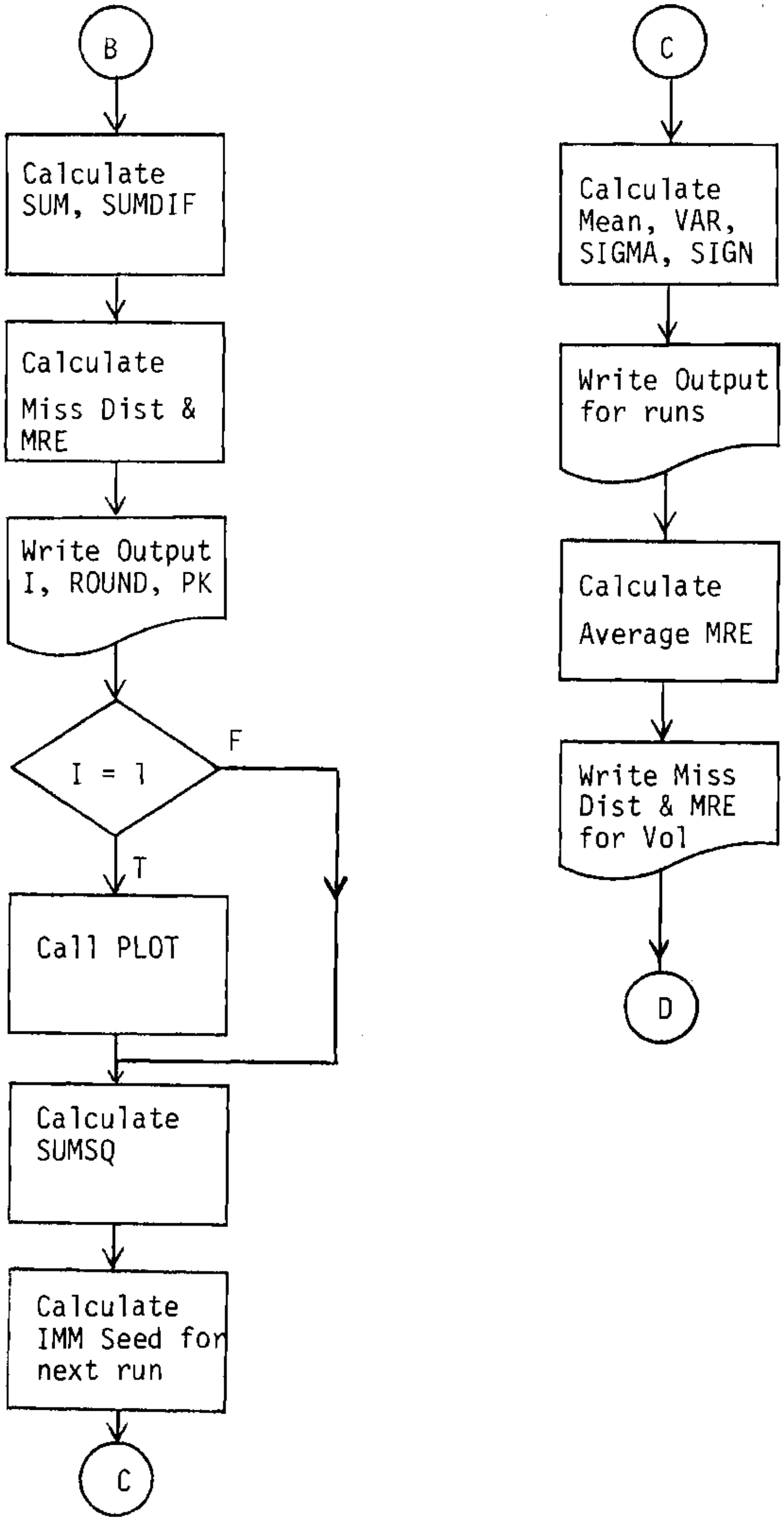


Figure E-2. Flowchart for Program.

```
      PROGRAM MAIN (INPUT,OUTPUT,TAPE1=INPUT,TAPE3=OUTPUT)
      DIMENSION XA(999),YA(999),U(5),V(5),L(5),XW(999),YW(999)
      DIMENSION XDIF(30),YDIF(30),MRE(30)
      REAL MEAN,MLR
      REAL MRE(30)
      MEAN=MM=RND=SIGMA=SIGN=VAR=0.0
      DO 100 I=1,999
  100 XA(I)=XW(I)=YA(I)=YW(I) = 0.0
      DO 101 J=1,20
  101 XDIF(J)=YDIF(J)=0.0
      S=1
    5 READ(1,1002)A,MLR,PC,X,Y,ORINT
      IF(EOF(1)) 1000,6
    6 READ(1,1003) SIGXT,SIGYT,SIGL,SIGA,XG,YG
      READ(1,1004) M,N,IMM
      IF(IMM.EQ.0)IMM=123456789+1000000.*RANF(S)
      B=ATAN2(YG-Y,XG-X)
      SUM=SUMSQ=SUMRD=0.0
      WRITE(3,1006) A,MLR,PC,B,SIGXT,SIGYT,SIGL,SIGA,XG,YG,M,N
      WRITE (3,1016) IMM
      WRITE(3,1011)X,Y,ORINT
      ORINT=ORINT/57.29578
      I=1
      Z2=(PC**2)*(SIGXT**2+SIGYT**2)
      MT=0
   90 K=0
      READ(1,1005)((L(NV),U(NV),V(NV)),NV=1,5)
   91 K=K+1
      IF(L(K).LT.0) GO TO 77
      MT=MT+L(K)
      NN=I+L(K)-1
      DO 92 NM=I,NN
      XA(NM)=X+U(K)*COS(B)-V(K)*SIN(B)
      YA(NM)=Y+U(K)*SIN(B)+V(K)*COS(B)
   92 WRITE(3,1012)U(K),V(K),XA(NM),YA(NM)
```

```
      I=NN+1
   93 IF(K.LT.5) GO TO 91
      GO TO 90
   77 DO 55 I=1,N
      CALL RANSET(IMM)
      PROD=1.0
      XM=X+SIGXT*RNORM(S)*COS(ORINT)-SIGYT*RNORM(S)*SIN(ORINT)
      YM=Y+SIGXT*RNORM(S)*SIN(ORINT)+SIGYT*RNORM(S)*COS(ORINT)
      IF(RANF(S).GT.PC) GO TO 7
      SIG=.2*SQRT((SIGXT**2+SIGYT**2)/2.0)
      XT=XM+SIG*RNORM(S)
      YT=YM+SIG*RNORM(S)
      GO TO 8
    7 XT=X+SIGXT*RNORM(S)*COS(ORINT)-SIGYT*RNORM(S)*SIN(ORINT)
      YT=Y+SIGXT*RNORM(S)*SIN(ORINT)+SIGYT*RNORM(S)*COS(ORINT)
    8 D2=(X-XM)**2+(Y-YM)**2
      LL=0
      IF(D2.GT.Z2) GO TO 10
      DO 15 J=1,M
      XA(J)=XM
   15 YA(J)=YM
      GO TO 26
   10 MM=(1-PC)*M
      CALL AIMPT(MM,MT,XA,YA)
      LL=MM+1
      DO 20 J=LL,M
      XA(J)=XM
   20 YA(J)=YM
   26 CONTINUE


      XBIG=YBIG=-500000.0
      XSMALL=YSMALL=500000.0
      ROUND=0.0
      DIST=0.0
      SXDIF = 0.0
```

```
      SYDIF = 0.0
      SUMMRE=0.0
      DO 51 J=1,M
      R=SQRT((XG-XA(J))**2+(YG-YA(J))**2)
      CTH=(XG-XA(J))/R
      STH=(YG-YA(J))/R
      XW(J)=XA(J)+SIGL*RNORM(S)*CTH-SIGA*RNORM(S)*STH
      YW(J)=YA(J)+SIGL*RNORM(-S)*STH+SIGA*RNORM(-S)*CTH
      XBIG=AMAX1(XBIG,XW(J))
      XSMALL=AMIN1(XSMALL,XW(J))
      YBIG=AMAX1(YBIG,YW(J))
      YSMALL=AMIN1(YSMALL,YW(J))
      CALL PKILL(XW(J),YW(J),XT,YT,A,MLR,J,PKT,RD)
      XDIF(J)=XW(J)-XT
      YDIF(J)=YW(J)-YT
   50 SXDIF = SXDIF+XDIF(J)
      SYDIF = SYDIF+YDIF(J)
      MRE(J)=SQRT(XDIF(J)**2.+YDIF(J)**2.)
      SUMMRE=SUMMRE+MRE(J)
      WRITE (3,1016) XDIF(J),YDIF(J),MRE(J)
      ROUND=ROUND+PKT
   51 PROD=PROD*(1.0-PKT)
      PK=1.0-PROD
      SUMRD=SUMRD+ROUND
      AXDIF = SXDIF/M
      AYDIF = SYDIF/M
      AVGMPI=SQRT(AYDIF**2+AXDIF**2)
      AVMRE=SUMMRE/M
      WRITE(3,1017) AVGMPI, AXDIF,AYDIF
      WRITE(3,1019) AVMRE
      WRITE(3,1015) I,RND,ROUND,PK
      IF(I.NE.1) GO TO 122
      CALL PLOT(M,XBIG,XSMALL,YBIG,YSMALL,XW,YW,X,Y,XT,YT)
  122 SUMSQ=SUMSQ+ROUND**2
      IMM=IMM+1000000.0*RANF(S)
      RND=RANF(S)
   55 CONTINUE
```

```
      MEAN=SUMRD/N
      IF(N.EQ.1) GO TO 56
      VAR=(SUMSQ-N*MEAN**2)/(N-1)
      GO TO 57
   56 VAR=(SUMSQ-N*MEAN**2)/(N)
   57 SIGMA=SQRT(VAR)
      SIGN=SIGMA/SQRT(FLOAT(N))
      WRITE(3,1008) MEAN,VAR,SIGMA,SIGN
      GO TO 5
 1000 STOP
 1001 FORMAT(1H1//10X,*FOR THIS RUN THE RANDOM SEED IS *,E16.10)
 1002 FORMAT(F4.1,F4.2,F4.2,3F7.2)
 1003 FORMAT(4F5.2,2F7.2)
 1004 FORMAT(2I5,I9)
 1005 FORMAT(5(I2,2F6.2))
 1006 FORMAT(1H1//10X,*PARAMETERS USED IN THIS RUN:*////10X,*A=*,F6.2,
     *    10X,*MER=*,F6.2,10X,*PC=*,F6.3,
     1    10X,*B=*,F9.3/10X,*SIGXT=*,F7.2,10X*SIGYT=*,F7.2,10X,*SIGL=*,
     2    F7.2,10X,*SIGA=*,F7.2/10X,*XG=*,F10.2,10X,*YG=*,F10.2,10X,*M=*,
     3    I7,10X,*N=*I7)
 1007 FORMAT(10X,*SAMPLE PT.*,I5,3X,*PK=*,F5.2,5X,*SUM=*,F7.2,5X,
     1    *SUMSQ=*,F10.2)
 1008 FORMAT(1X//10X,*SAMPLE RESULTS:*/10X,*MEAN=*,F7.4,5X,*VAR=*,F8.2,
     1    5X,*SIGMA=*,F7.4,5X,*SIGMA/SQRT(N)=*,F10.4)
 1009 FORMAT(50(10X,*AIM PT. NO. *,I4,3X,*XA=*,F8.2,5X,*YA=*,F8.2/),1H1)
 1010 FORMAT(10X,*MICS. DATA:*,4(5X, F7.3))
 1011 FORMAT(10X,*TARGET EST=*,2(F9.2,3X),*TA ORINT=*,F7.2,/)
 1012 FORMAT (10X,*U=*,F9.2,3X,*V=*,F9.2,3X,*XA=*,F9.2,3X,*YA=*,F9.2)
 1013 FORMAT(6X,*X(ROUND)*,4X,*Y(ROUND)*,4X,*ROUND*,3X,*X(TARGET)*,3X,*Y
     4(TARGET)*,4X,*PKILL*)
 1014 FORMAT(2F9.2,I5,2F9.2,F5.3)
 1015 FORMAT (10X,*DRAW=*,I5,5X,*RND=*,F9.7,5X,*ROUND=*,F9.2,3X,*PK=*,F
     *5.3,/)
 1016 FORMAT(10X,*ISEED = *,I9,/)
 1017 FORMAT(/10X,*MISS DIST=*,F7.2,4X,*XDIF=*,F7.2,4X,*YDIF=*,F7.2)
 1018 FORMAT(/10X,* XDIF(J)= *,F10.2,* YDIF(J)= *,F10.2,* MRE(J)= *,F10.
     12)
```

```
1019 FORMAT(/10X,*AVERAGE MRE = *,F12.2)
     END


     FUNCTION RNORM(S)
     DATA PI/3.14159/
     IF(S.LT.0.0) A=SIN(2.0*PI*RANF(ABS(S)))
     IF(S.GE.0.0) A=COS(2.0*PI*RANF(S))
     RNORM=SQRT(-2.0*ALOG(RANF(ABS(S))))*A
     RETURN
     END


     SUBROUTINE AIMPT(MM,MT,XA,YA)
     DIMENSION XA(999),YA(999)
  95 DO 100 I=MT,MM,MT
     DO 100 J=1,MT
     XA(J+I)=XA(J)
 100 YA(J+I)=YA(J)
     RETURN
     END


     SUBROUTINE PKILL(XW,YW,XT,YT,A,MLR,J,PKT,RD)
     REAL MLR
     IF(MLR.LE.0.0)GO TO 40
     B=.69315/MLR
  40 RD=SQRT((XW-XT)**2+(YW-YT)**2)
     TEMP=RD-A
```

```
      IF(TEMP.GE.(3*MLR)) GO TO 42
      IF(TEMP.LE.MLR) GO TO 41
      PKT=0.5*(1.0-(TEMP-MLR)/(2.0*MLR))
      GO TO 50
   41 IF(TEMP.LE.0.0) GO TO 45
      PKT=EXP(-B*TEMP)
      GO TO 50
   42 PKT=0.0
      GO TO 50
   45 PKT=1.0
   50 CONTINUE
      RETURN
      END


      SUBROUTINE PLOT(NPTS,XBIG,XSMALL,YBIG,YSMALL,XW,YW,X,Y,XT,YT)
      DIMENSION XW(NPTS),YW(NPTS),PRINT(131)
      JLAST=JL1=TEMP=YJ=YMIN=0.0
      DATA BLANK,CHAR,WHY,EX/1H ,1H*,1HY,1HX/
      XSIDE=XBIG-XSMALL
      YSIDE=YBIG-YSMALL
      XSPACE=XSIDE/65.
      YSPACE=XSPACE/0.3
      IF(56.*YSPACE .GE. YSIDE) GO TO 10
      YSPACE=YSIDE/56.
      XSPACE=.6*YSPACE
   10 YSPACE=YSPACE*1.25
      XSPACE=XSPACE*1.25
      SCALE=10.*XSPACE
      WRITE(3,20)X,Y,SCALE,XT,YT
   20 FORMAT(*1(X,Y) = (*,1PE15.4,*,*,E15.4,*).  ONE INCH = *,E15.4,0P
     1      * METERS.*,*(XT,YT)=  (*,F10.2,*,*,F10.2,*)*)
C     ORDER XW AND YW ARRAYS IN DECREASING Y ORDER
      NPTS1=NPTS-1
      DO 1 I=1,NPTS1
```

```
      K=NPTS-I
      DO 2 J=1,K
      IF(YW(J) .GE. YW(J+1)) GO TO 2
      TEMP=YW(J+1)
      YW(J+1)=YW(J)
      YW(J)=TEMP
      TEMP=XW(J+1)
      XW(J+1)=XW(J)
      XW(J)=TEMP
    2 CONTINUE
    1 CONTINUE
      JLAST=0
      DO 3 J=1,57
      YJ=29-J
      DO 4 I=1,131
    4 PRINT(I)=BLANK
      IF(YJ .NE. 0.) GO TO 6
      DO 5 I=1,131
    5 PRINT(I)=EX
    6 PRINT(66)=WHY
      YMIN=(YJ-.5)*YSPACE+Y
      IF(JLAST.EQ.NPTS) GO TO 9
      JL1=JLAST+1
      DO 7 K=JL1,NPTS
      IF(YW(K) .LT. YMIN) GO TO 8
      I=(XW(K) - X)/XSPACE + 66.5
    7 PRINT(I)=CHAR
      JLAST=NPTS
      GO TO 9
    8 JLAST=K-1
    9 WRITE(3,30) PRINT
   30 FORMAT(1X,131A1)
    3 CONTINUE
      WRITE(3,50)
   50 FORMAT(1H1)
      RETURN
      END
```

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